COLLECTIVE COHERENT OSCILLATION PLASMA MODES IN SURROUNDING MEDIA OF BLACK HOLES AND VACUUM STRUCTURE - QUANTUM PROCESSES WITH CONSIDERATIONS OF SPACETIME TORQUE AND CORIOLIS FORCES

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Abstract. The main forces driving black holes, neutron stars, pulsars, quasars, and supernovae dynamics have certain commonality to the mechanisms of less tumultuous systems such as galaxies, stellar and planetary dynamics. They involve gravity, electromagnetic, and single and collective particle processes. We examine the collective coherent structures of plasma and their interactions with the vacuum. In this paper we present a balance equation and, in particular, the balance between extremely collapsing gravitational systems and their surrounding energetic plasma media. Of particular interest is the dynamics of the plasma media, the structure of the vacuum, and the coupling of electromagnetic and gravitational forces with the inclusion of torque and Coriolis phenomena as described by the Haramein-Rauscher solution to Einstein's field equations. The exotic nature of complex black holes involves not only the black hole itself but the surrounding plasma media. The main forces involved are intense gravitational collapsing forces, powerful electromagnetic fields, charge, and spin angular momentum. We find soliton or magneto-acoustic plasma solutions to the relativistic Vlasov equations solved in the vicinity of black hole ergospheres. Collective phonon or plasmon states of plasma fields are given. We utilize the Hamiltonian formalism to describe the collective states of matter and the dynamic processes within plasma allowing us to deduce a possible polarized vacuum structure and a unified physics.

I. INTRODUCTION

In this paper we present a generalized model of the balance between the gravitational and electromagnetic fields near or at the ergosphere of a black hole. A. Einstein, [1] J. A. Wheeler [2] and many other researchers have attempted to reduce both gravitation and electromagnetism concepts to the principles of geometry. As is well known, the geometrization of gravity has met with great success, while the latter endeavor for electromagnetism has met with many difficulties. In the case of a black hole, the charge of the heavier ions, by charge separation will be closer to the ergosphere than the negative ions or electrons. Electric field polarization will occur by its emission from the rotating body or system. Magnetism will arise in the vacuum induced by polarization by the rotation of a gravitational body such as a pulsar or black hole. This model and the general interaction between electromagnetism and gravity is basic and involves the details of many-body physics and the structure of the vacuum. The vacuum is a potential source of electrons, positrons as well as other particles when activated by a polarizing energy source [3].

Our new and unique approach of developing the relativistic Vlasov equation, formulated and solved in the vicinity of black holes does, indeed, describe the electromagnetic phenomena of a dense plasma under a strong gravitational field. In the extreme gravitational conditions in a black hole, photons are trapped by being strongly bent by the gravitational field described by the curvature of space. Interaction between the media outside and the inside of a black hole can occur due to vacuum state polarization i.e. the properties of the vacuum, angular momentum of the black hole (Kerr metric) and charged (Kerr-Newman metric) as well as magnetic field coupling through plasma vacuum state polarization.

The vacuum rotating gravitational field gives rise to electromagnetic forces which are given by

1. $$B \propto \left(\frac{e}{c^3}\right)\underline{g} \times \underline{\omega}$$

where e is the charge on the electron, c is the velocity of light, $\underline{g}$ is the local gravitational acceleration, and $\underline{\omega}$ is the angular velocity of rotation of the body or black hole. The term $\underline{g} \times \underline{\omega}$ is analogous to a gravitational gyroscopic term. If v_{Esc} is the escape velocity of an electron on the event horizon of a black hole then $v_{Esc} \sim c$. The highly bent space of a black hole generates a higher magnetic and charge field often observed near a pulsar.

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In a black hole, gravity is so strong that space is so sharply curved that the gas of the interstellar media is compressed and becomes dense, and like any hot gas, emits radiation in the form of radio waves, visible light, and x-rays. This electromagnetic field effect across the event horizon acting through the effects of vacuum state polarization correlates external and internal effects and hence may resolve the information paradox so that information going into a black hole is conserved with charge, angular momentum and information is transformed by the black hole. Black holes act as an electric generator power source of quasars which emit the light of an entire galaxy. Of course, the black hole stores energy from the gravitational field and, as R. Penrose suggested, also stores a great deal of energy in its rotation. As further collapse occurs, more energy is generated to power the quasar [3].

The plasma dynamics in the external region generates electric field gradients and hence current flow and induces intense magnetic fields across the ergosphere. The event horizon is stretched and acts as a conducting sphere with a resistivity, for example, having an impedance of 377 Ω. Magnetic lines of force pass across the sphere, exciting its surface with eddy currents producing drag on the sphere. The lines of force do not cross the horizon but wrap around it and, for a rotating system, they eventually pinch off as loops. Astrophysical effects on the black holes occur through the effects of their excited states of the dense plasma on the vacuum. For 377 Ω, an electric field of 377 volts would be needed to drive one ampere of current across a square surface area on the event horizon. This value is chosen, for the sake of this picture, analogous to the Earth's fields. It is of interest to note that the magnetohydrodynamics and Coriolis forces of the plasma's collective behaviors in this picture are similar to the process of sunspot formation and coronal ejection on our sun. Thereafter, close examination of black holes ergospheres structures may reveal regions of high magnetic flux and x-ray emissions resembling the sunspot activity found on our local star.

Of course, the motion of the magnetic field by the dynamic processes near a black hole generates an electric field which can give us a quantitative method to describe the energy transfer mechanisms. In the case of a rapidly rotating magnetized black hole, the electric field generated near the event horizon can produce enormous voltage differences between the poles of the spinning body and its equatorial region. As much as 10^{20} volts may be generated through field lines stretched at the event horizon, resulting in the system acting as an enormous battery. The magnetic field lines carry current which are driven by the voltage difference to distant parts of a quasar, which are linked by the magnetic field lines and the vacuum state polarization in its environment, producing a gigantic direct current circuit. Positive charges flow up the field lines from the equatorial regions of the surface and are balanced by the current from the polar field lines to the equatorial lines. The complex properties of the energized plasma feeds the jets of ionized gases that have been observed emerging from the nuclei of quasars, supernovae and galaxies, stretching out many light years into space. The plasma can act as if it is frozen around magnetic field lines, where the electrons undergo gyroscopic spin. As the lines of magnetic force thread through the ergosphere, energy is deposited in the intervening plasma, accelerating it outward against the strong magnetic field. This process is balanced by the pull of gravity in the vacuum of the black hole's event horizon. Hence a balance is maintained at certain phases of collapse stability, where energy balance occurs.

The processes of plasma magneto-electrodynamics with a large magnetic field in the strong gravitational field of a black hole act as a generator/magnetic motor. The generated Coriolis forces in the plasma media occur due to the rotational acceleration as well as the gravitational field of the black hole. As we demonstrated in detail, the angular momentum properties result from the torque term in Einstein's stress-energy tensor [4]. The resulting acceleration produces electromagnetic biases in the electron-positron states in the vacuum producing the polarization of the vacuum which we demonstrate here and in reference [5]. This requires that we include the magnetic field in the Vlasov equation [6]. It is the strong magnetic field case that gives us the dynamo generator dynamics displayed by galactic and supernovae black holes. Shockwave and bow wave phenomena can occur because of violent plasma eruptions in a strong magnetic field and bow wave phenomena can occur when the black hole is associated with a second astrophysical body in which the two exchange magnetic lines of flux and plasma fields [7].

We and others have described elsewhere the manner in which the strong force and the gravitational forces can become balanced through the formalism of the relationship of quantum chromodynamics (QCD) and quantum electrodynamics (QED). The strong and electroweak forces are related through the quark model. This model utilizes the existence of mini Planck unit black holes [8]. Thus we can describe the form of the dynamics of the plasma energy tensor by treating its effect through the Coriolis forces. These accelerative driving forces activate the plasma dynamics and, hence the effect of the vacuum is manifest through the effect of the torque term in the stress-energy tensor. This is the manner in which the stress-energy tensor is modified which we detailed in references [3,4]. Hence the torque term in the stress-energy tensor actually yields the more detailed and accurate Einstein-Vlasov model because plasma can be utilized in this approach [9,10].

These turbulent perturbations often diffuse and propagate transverse to the magnetic lines of force. Many higher order terms and a number of coupling constants are not directly amenable to an analytic approach and require computer simulations. Under such variable gravitational and electromagnetic conditions, patterns can emerge under cyclical interactions but also large dynamical unpredictable instabilities will occur. Our wave equations must accommodate these two cases. Some of the more detailed analytic approaches can be found in reference [5]. We describe examples of black hole plasma systems for stellar, and supernovae phenomena. In this paper, we express in detail the balance equations between the gravitational collapsing system and the surrounding plasma. Balance systems act in a thermo-plasma-gravitationally coupled systems that obey unique structures in space, some of which we present in this volume.

We can treat the electromagnetic field in terms of spherical harmonics as an approximation. We have solved Einstein's field-curvature equation with a centrifugal term that arises out of the torque term in the stress-energy tensor term, and source term and demonstrate a possible balance equation at the event horizon [3,4]. The high magnetic field of neutron stars of about 10^{14} Gauss, and possibly the black holes also act to direct and repel the plasma against accretion at the event horizon surface. We find soliton or magneto–acoustic plasma states as solutions to the relativistic Vlasov plasma equations solved in the vicinity of a black hole ergosphere.

II. DYNAMIC TURBULENCE AT THE SURFACE OF THE EVENT HORIZON AND THE BALANCE EQUATION

It is clear that the interface at the surface of a rotating neutron star, pulsar or black hole and the surrounding media can be highly turbulent. Large energy, thermal, charge, matter and angular momentum charges occur. Excitation modes in the plasma media can become quite large. For small excitations, the standard approach is to decompose the turbulent modes into a sum of linear modes, but that is not possible in our case because the system is so nonlinear.

In high excitation phenomena, which exceed the thermal energy, the nonlinearity condition requires that the various modes of excitation couple with each other in a variety of ways. Let us consider an example of a wave resonant mode coupling for wave vectors $\underline{k} \equiv 1/\lambda$ where λ is the wavelength and wave amplitude $U(k)$. The condition for the wave resonant mode coupling we can express as $\omega(\underline{k}-\underline{k}')+\omega(\underline{k}')=\omega(\underline{k})$ having a dominant frequency, ω which is satisfied for wave vectors, $\underline{k}$ and $\underline{k}'$.

Turbulence theory yields a nonlinear wave kinetic equation of the form for the wave amplitude

$$\text{2.} \qquad \frac{\partial |U(\underline{k})|^2}{\partial t} = 2\gamma(\underline{k})|U(\underline{k})|^2 + \sum_{k'} A\left(\underline{k},\underline{k}'\right)|U(k-k')|^2|U(k')|^2 + \sum_{k'} B(k,k')|U(\underline{k})|^2|U(k')|^2$$

where A (k,k') and B(k,k') are the coupling coefficients which describe resonant and non-resonant modes of the coupling processes, respectively, and the coefficient γ is the linear growth rate. For $\gamma/\omega << 1$ we have the weak turbulence theory. In our case we will be dealing with the high turbulence theory $\gamma/\omega >> 1$ which carries more coupling terms and hence is more complex.

Properties of the media of the plasma in the balance equation occur at the approximate region of the event horizon. Our balance force equation for black hole dynamics, in complex interactions, relates the gravitational and electromagnetic force. The dominant force is the major attractive force toward gravitational collapse. Opposing forces exist for the Kerr-Newman system in which rotational centrifugal and Coriolis forces are driven by spin and charged particles dynamics and the torque term in Einstein's stress-energy tensor. In general, we will not concern ourselves with individual particle interactions and deal primarily with collective particle dynamics. Although these collective particle processes arise out of individual particles and their mass action, currently, much is known about their mass action, and we can utilize these formulations for our present purpose.

Dynamic black hole physics involves thermodynamic processes as well as electrodynamic and gravitational collapse phenomena. In considering the Kerr and Kerr-Newman solutions, we can address the concept of radiated and absorbed energy in a collapsing system. If a superdense star or stellar cluster is collapsing, rotating and is charged, the possibilities of complex matter near the black hole is much more complicated and hence is a much more interesting dynamic system. In general, such a system is much more observable, as an x-ray and visible source, because a finite rotating event horizon exists along with a "tidally acting" ergosphere.

In general, it is considered that the net charge on stellar and galactic collapsing systems is relatively small but extreme internal charge separation can occur. The major phenomena, however, is the rotation of the system, hence the Kerr solution is often utilized. The angular momentum of the system generates Coriolis type forces and these types of forces drive convective currents. Some examples on earth are the ocean and ionospheric currents as well as magnetospheric dynamics. Also, sunspot migration is affected by Coriolis-like tides and plasma phenomena. This too occurs in stellar and quasar structures. Similar type forces can drive stellar matter plasmons near black holes resulting from ergospheric tidal action. Patterns of material and current flow can occur over Northern and Southern hemispheres which may be necked (or pinched) at the equator [6]. Radiative processes can be expressed by the Stephan-Boltzmann equation, where the energy is related to the temperature as $\mathrm{E} = a\mathrm{T}^4$ and

3. $$a = \frac{3}{5}\pi^4 \frac{R}{v^3}$$

where R is the gas constant and v is the frequency. The Boltzmann constant is $K = R / A$ where A is Avogadro's number [7].

The torque term in Einstein's field equations generates rotations and spin driving forces such as the Coriolis forces. Analysis of the spin effects of the black hole is key to the understanding the surrounding plasma dynamics. Centrifugal and Coriolis forces in the plane of rotation affect the surrounding plasma spin effects, expelling the plasma, opposing the accretion process near the event horizon [8]. The detailed role of these forces requires extensive computer modeling. Progress has been made by Feder and others [9]. These driving forces can be augmented by large magnetic fields as well as the strong attractive forces of ultra dense matter of the black hole. These systems are comprised of the rotating black hole and its surrounding plasma gas media. We can form a crude analogy to the ionospheric media surrounding the earth, its gravitational field and "steady state" magnetic field. The charged ionospheric and magnetospheric layers are affected by these forces, in addition to the temperature differential from equator to poles, and under seasonal variations. Coriolis forces and convective currents are driven from west to east in circulating loops [6]. Solar wind activity also acts as an external driving force and although these patterns are complex, they are statistically approximately repeatable. Similar processes can be applied to the solar and stellar dynamics surrounding media composed of energetic plasma. The outermost loops are driven by centripetal (gravitational) and centrifugal (rotational) forces.

III. THE BALANCE EQUATION IN THE VICINITY OF A BLACK HOLE ERGOSPHERE

A black hole system undergoing collapse in a charged rotating system is surrounded by a plasma field. A balance between the energetic plasma field and the gravitational forces exists. As the gravitational collapse moves inward to the black hole center, the surrounding plasma, through its magnetic stress field, repels from its black hole event horizon. Furthermore, the springiness and elasticity of the magnetic lines of force in the excited plasma states is caused by the centrifugal rotational and Coriolis forces, balanced by the gravitational collapsing forces.

Hence we can introduce the gravitational force in the Vlasov equation to balance and repel the electromagnetic force. The curl of the field gives a rotational component and the plasma field is fully charged so that we must consider a Kerr-Newman rotating, charged black hole system. It is the plasma field excitation modes that make collapsing black holes visible and hence detectable.

We develop a modified form of the Vlasov equation in a gravitational field. From this formalism, we develop a balance equation. We find solutions to our modified Vlasov equation which describe coherent, collective states that polarize the vacuum and hence form a preferred direction in space. This picture relates to our spacetime torque and modified spin model of the Haramein-Rauscher solution [3]. Preferred directions in space are not precluded by the structure of Einstein's field equations but were thought by Einstein not to exist. Mach's Principle, however, may yield clues in regard to a preferred reference field or frame. We address this issue in more detail later in this paper.

We detail the formalism of the balance equations including the thermodynamics of black hole physics and external black hole plasma dynamics. Radiative Stephan-Boltzmann terms and convective rotational motion is considered as well as conductive properties of the plasma. These properties are significantly affected by the nonlinear properties of the media and the polarization of the vacuum. Consider a nonlinear, coherent, collective phonon or plasmon state in a plasma field. This field is described by the solution of a nonlinear form of the dynamical Vlasov equation. These solutions relate to the coherent states, are soliton like, and are observed as phonons.

The Vlasov equation describes the plasma state of a fully ionized gas in an electromagnetic field. Essentially these conditions are a specialized and extended case which has parameters not described by Maxwell's equations alone (Maxwell's formalism does not deal with the nonlinear gas dynamics of a fully ionized plasma and non-Hertzian wave phenomenon). The function that is a solution of the Vlasov equation is expressed as a distribution function, f_i for species i which is a function of space, momentum and time. The equation of Boltzmann and the Fokker-Planck in phase space are kinetic equations [10]. The kinetic equation is a self-contained equation for the distribution function. The Fokker-Planck coefficient terms express and reproduce the Balescu-Lenard collective collision terms giving us an expression for collision effects [10]. The usefulness of this approach was shown by Vlasov [11].

Our balance force equation for black hole dynamics relates the gravitational and electromagnetic forces. The dominant forces are the major attractive forces towards gravitational collapse. The opposing forces exist for the Kerr-Newman system in which rotational centrifugal forces are driven by spin and charged particle dynamics. In general, we will not concern ourselves with individual particle interactions and deal with collective particle dynamics primarily. Although these collective particle processes arise out of individual particles and their mass action, and since, currently, much is known about their mass action, we can utilize these formulations for our present purpose. Note that both transverse and longitudinal modes of excitation in the plasma are possible.

We can derive the equilibrium state of the balance equation from the kinetic equation for the plasma using the Fokker-Planck equation in momentum space. Quantum kinetic properties can be included in this formalism for a system of particles with Coulomb interaction which was derived by Landau from the Boltzmann equation [12]. The Debye length sets a limit on the distance correlation of particles and is described by a system formulated in terms of a series to have a cut off and to diverge to infinity and hence the Debye length acts as a cut off approximation to avoid nonrenormalization. The divergencies over long distance excitation of which we primarily deal with, in a plasma, are longitudinal or acoustic or plasmon modes.

For a homogeneous distribution of charged particles in a plasma, most oscillation is produced by the light mass charged electrons of the plasma. For diffusion and dynamic friction we can write the equation

4. $$f_i \text{ as } \frac{\partial f_i}{\partial t}(p,t) = \sum_{\mu\nu} \frac{\partial \mathrm{B}_{\mu\nu}}{\partial p_\mu} \frac{\partial}{\partial p_\nu} f_i + \frac{\partial}{\partial p} \bullet \underline{\mathrm{A}} f_i$$

where, in this specific case, $\mu, \nu = 1,2,3$ and where $B_{\mu\nu}$ and $\underline{A}$ are respectively the coefficients of diffusion and the coefficients of dynamic friction. The friction concept is part of the balance equation dynamic. Both of these coefficients can be written in two parts, one for large and one for small energies of charged particles. The expressions for these two coefficients for $B_{\mu\nu}^{Vib.}$ and $\underline{A}^{Vib.}$ as $B_{\mu\nu} = B_{\mu\nu}^{Coll.} + B_{\mu\nu}^{Vib.}$ and $\underline{A} = A^{Coll.} + A^{Vib.}$ where "Coll." stands for collective and "Vib." for individual vibrations

5. $$B_{\mu\nu}^{Vib.} = \frac{KT}{2\pi} \int \delta\left(\omega_L - \frac{\underline{k} \bullet \underline{p}}{m}\right) a_{k\mu} a_{k\nu} d\underline{k} \, \underline{A}^{Vib.} = \frac{e^2}{2\pi m} \int \delta\left(\omega_L - \frac{\underline{k} \bullet \underline{p}}{m}\right) a_k \left(a_k \bullet \underline{p}\right) d\underline{k}$$

where $\underline{p}$ is the momentum, t is the time, and k is the wave number $k = {}^{2\pi}\!/_{\lambda}$, and ω_L is the Langmuir frequency given as $\omega_L = \sqrt{4\pi e^2 n / m}$, and K is the Boltzmann constant. The quantities $B_{\mu\nu}^{Vib.}$ and $\underline{A}^{Vib.}$ are $\neq 0$ only when $v > c$ in a media so that Cherenkov radiation exists in a longitudinal plasma for $p/m > \omega_L / k$ and m is the mass of the particle that is excited. Only plasma waves having a wave number $k > \omega_L m / p$ can be excited as a result of deceleration of the electrons with momentum, p. The maximum value of the wave number is determined by the magnitude of the Debye radius $r_D = \sqrt{KT / 4\pi e^2 n}$. The deceleration of the electron due to radiation of the longitudinal waves is possible only under the condition that their velocity is higher than the mean thermal velocity. The dynamically active particles of the media are the lighter electrons, rather than the ions.

Integration over wave numbers $d\underline{k}$ along the motion and using only terms in $\mu = \nu$ as taken to be different from zero we have the following expressions for the diffusion coefficients. We have

6. $$B_{33}^{Vib.} = \frac{e^2 KT}{v^3} \omega_L{}^2 \ln \frac{v}{v_T \pi} \text{ and } B_{11}^{Vib.} = B_{22}^{Vib.} = \frac{m e^2 \omega_L{}^2}{2v} \text{ for } v = {}^{p}\!/_{m}$$

and $\eta, \nu = 1$ is analogous to the x coordinate, $\mu, \nu = 2$ is analogous the y coordinate, and $\mu, \nu = 3$ is analogous to the z coordinate. We express the deceleration force, F, acting on a charged particle due to longitudinal waves so that upon integration of the equation for the magnetic field, B, as $F = \frac{e^2 \omega_L{}^2}{v^2} \ln \frac{v}{v_T}$; the friction force $F^{Vib.}$ is the same order of magnitude as $F^{Coll.} = \frac{e^2 \omega_L{}^2}{v^2} \ln \frac{r_D}{a}$ where $a \equiv e^2 / mv^2$.

In terms of thermodynamics the equations in terms of f_i and F_i for species v_i, the electron is essential to assure that the plasma vibrations or plasmons and the plasma particles surrounding the specific particles are initially in the state of thermodynamic equilibrium. This point is a key in that what we must construct is a thermodynamic equilibrium between the fully ionized electric charge dynamics and the gravitational attraction of the black holes, and we must construct a collective vibrating medium which comprises a self-consistent field. For long waves, the damping is small, which occurs in the low frequency range. Both longitudinal and transverse components exist and their relative significance depends on a number of factors such as the temperature, pressure, density and degree of ionization of the media as well as externally applied and internally generated fields and their coupling. Ion and electron properties such as pressure, dielectric constant and conductivity may be different under the conditions where quantum electron interactions occur [5]. Polar and nonpolar neutral members may also be present.

At sufficiently large wavelength and low frequencies, as in interstellar and stellar regions, there are longitudinal vibrations of the electron gas. In these frequency regions, we find that there are similarities between these vibrational spectra of the quantum plasmas and our own ionosphere. This is the case where the Fermi energy, ε_F, effects dominate the plasma rather than the temperature. Near the horizon temperature effects dominate in more energetic processes.

IV. PLASMA COHERENT EXCITATION AND THE VACUUM STRUCTURE

Our work may provide a new picture of a structured vacuum which relates to single particle and collective coherent particle state interactions. This active plasma field and its electromagnetic properties are in balance in the gravitationally collapsing process in and near a black hole. We will detail these processes in terms of first, the quantum electrodynamics of dense plasmas, second, the intense relativistic gravitational field near a black hole event horizon, and third, the radioactive fields and other thermodynamic properties of black hole, supernovae, pulsar and quasar phenomena.

We consider the properties of dense plasmas in the vicinity of strong gravitational fields and their collective coherent states. We must include particle-particle and particle-field coupling in our eventual formulation of a metrical space and stress-energy tensor in Einstein's field equations. Work has been conducted on the Einstein-Vlasov equations [13] which is a good start but does not include the many body processes of dense plasmas including the effects of particle-particle, particle-field and particle-field coupling to the structure of the vacuum plus self energy states. Through this picture, the properties of a structured vacuum will emerge and hence, we can understand the fundamental role of the vacuum in forming and shaping these processes.

In the region near the outside of the event horizon of a black hole, we can no longer consider the approximation of a collisionless plasma. This approximation is usually made for describing man-made and natural non-dense plasma phenomena. When collisions are taken into account, the problem becomes more complex but more interesting. We must include quantum interactions and vacuum state polarization in this many-body problem [5]. Superdense, fully ionized plasmas occur where we have strong gravitational forces surrounding, and in, a black hole dynamical system. Although the plasma media is fully ionized, such a system has been termed a "solid-state" plasma where an analogy is made between plasmon and photon collective oscillations of the plasma media [5,14]. Near the event horizon, the plasma is superdense where quantum effects occur. The plasma-particle interaction must be properly treated quantum mechanically when the electron plasma-wave phonon energies are comparable to, or greater than, the mean random electron energies, and/or, when the phonon momenta are of the order of magnitude or greater than the average electron momenta in the plasma. This will lead to a new formulation of quantum gravity.

Whether classical or quantum plasma treatment is considered, the collective properties, as well as the single-particle properties must be considered. The collective properties of the plasma become important when it interacts with an external or self-generated radiation field. This occurs in the case where the electron plasma frequency, ω_ρ, is of the same order of magnitude, or exceeds, the operating radiation frequency ω, i.e. $\omega_\rho \geq \omega$. The value of ω_ρ is of the order of 10^5 Hz or greater. A plasmon is defined as a collective mode of oscillation of a plasma gas and a phonon is defined as a collective mode of oscillation of a solid such as a crystal lattice and usually associated with acoustics. The solid state high density plasma systems have both plasmon and phonon collective modes of oscillation. External and self-generated electromagnetic fields can also act to produce excitation modes. In a sense, the phonon in a superdense solid state plasma acts like a charge separated phonon made in a crystal. The criteria that distinguish the properties of a plasma, as to whether it is classical or quantum mechanical in nature, can be defined in terms of three fundamental lengths of the electron gas. These definitions hold for the first approximation of a one component plasma and are the classical length, βe^2, the Debye screening length, $\lambda_D = \left(4\pi\beta e^2 / \rho\right)^{-1/2}$, and the thermal deBroglie wavelength, $\lambda = \hbar(\beta / 2m)^{-1/2}$, for β defined as $1/KT$, where K is the Boltzmann constant. From these three quantities, we can define two dimensionless parameters. They are the classical parameter $A = {\beta e^2}/{\lambda_D}$ and the quantum parameter $\delta = \lambda / \lambda_D$ which is a measure of the existence of quantum effects. For a quantum plasma $\delta > 1$, and in the classical limit, $(h = 0), \delta = 0, A < 1$. We must also take into account the collective behavior characterized by the plasma oscillations since charge screening effects are an automatic aspect of the electron plasma gas. We compare the plasma properties for the usual classical limit to that of a high density plasma in the quantum limit. This is appropriate for the problem we are addressing of a plasma field surrounding a black hole.

A. Plasma Oscillations and a Description of Collective Behaviours

The collective behavior of electrons was developed by Bohm and Pines and both the classical and quantum mechanical treatments were given [15]. The organized behavior of a high-density electron gas results in what is termed "plasma oscillations" and is treated by use of the collective description [16]. As opposed to the usual single-particle formulation, the collective model describes the long-range correlations in electron positions as a consequence of their mutual interactions. The collective modes of the plasma oscillations are called phonons or plasmons.

The self-consistent field methods of Hartree and Fock [17] neglect the long-range Coulomb forces and hence are not adequate for cases in which there exist high particle densities where electron-electron interactions become important. The plasma oscillations come about through the effects of long-range correlation of electron-positron pairs due to Coulomb interactions [18]. In the treatment of plasmons one considers a particular Fourier component of the average field as proportional to exp $\{-i(\underline{k} \cdot \underline{r} - \omega t)\}$. For small amplitudes, a linear expansion is valid. The condition for oscillations to continue to occur is that the field arising from the particle response must be consistent in phase with the field producing the response.

There are certain limitations on the collective description of the electron gas in terms of organized longitudinal oscillations due to the fact that these oscillations cannot be sustained for wavelengths shorter than the fundamental Debye screening length, λ_D. This critical distance can be expressed in terms of the distance, with a mean thermal speed, v, traveled during a period of one oscillation:

7. $$\lambda_D = \left(4\pi\beta e^2 / \rho\right)^{-1/2} \approx \bar{v} / \omega_\rho$$

For longitudinal waves, the approximate dispersion relation, for long wavelengths and small frequency [14,15] is

8. $$\omega^2 = \frac{4\pi\rho e^2}{m_e} + \frac{3k^2}{m_e\beta}$$

where ω is the frequency of an imposed uniform electric field, k is the wave-number $k \equiv 2\pi / \lambda$, where λ is the wavelength, ρ is the electron density, $\beta = 1/KT$ for K, the Boltzmann constant, and T the Kelvin temperature.

B. The Many-Body Problem and the Soliton Model of Plasma in Collective, Nonlocal and Coherent States

At a temperature of the order of 10^4 to 10^5 $^\circ K$, in a non-fully ionized plasma, energy will be transferred to neutral gas particles through elastic collisions. If the plasma is subjected to an externally varying electric field, an acoustic wave is generated in the neutral gas. Also, if the electric field is held constant, the electron density can be varied by an externally applied acoustic or sound-like wave. When the applied frequency of the plasma parameters are held in a proper relationship, a coupling of the electron energy to the acoustic wave can occur and can create a positive feedback amplification which results in acoustic-like waves manifesting as oscillations. We define these specific states as an "acouston". Acoustons can carry charge, unlike phonons. Sometimes these excitations are termed excitons. This can be the case for both internally and externally generated acoustic or acoustic-plasmon states [5,6].

Examination of these acoustic-plasmon or acouston growth modes and collective states that result from such an amplification are important in determining the conditions for spontaneous excitation of a normal mode of vibration in a plasma system. The electron density is key to the determination of the acoustic pressure field because of the coupling of the electrons to the neutral gas in the case of cooler plasmas. The speed of this ion-acoustic longitudinal wave is determined by the inertia of the ions and the "elasticity" of the electrons. In the presence of a magnetic and gravitational field, the plasma becomes non-isotropic and non-homogeneous.

It is through nonlinear pulsed electric and magnetic fields, timed at precisely pulsed non-uniform modes, which either enhance or diminish the growth of these acoustic modes. It is obvious that the collective plasma behavior is the mechanism for plasma collective state formation and hence, these modes can be enhanced or diminished by the form of the external or internally generated electric and magnetic fields and the geometric configurations. All these factors occur optimally to generate the dynamo effect in black holes which involve the collective nonlinear processes within the plasma. Growth of the so-called plasma instabilities, which we identify with a coherent soliton state, convert forms of energy from externally applied fields into coherent charged plasmon excitations [5]. Debye demonstrates that the thermal vibrations of a crystal lattice can be considered as traveling acoustic waves and that the transport properties of a metal, such as with electrical and thermal conductivity, are governed by the scattering of electrons from these vibrations. Also sound waves in a solid can be scattered by electrons. This is basic to the Vlasov model.

The lepton number for an electron in its lowest quantum state in the geometry of the gravitational force of a black hole can act as a ground state in the dynamics of the Freidman universe derived from the Schwarzschild lattice universe [19]. This model derives its origin from solid state physics. The dynamics of particles and fields is expressed for the Schwarzschild geometric condition. From this simple picture, the entire dynamics of the closed three-sphere lattice universe can be used to describe the Friedman model. We detail the Lindquist-Wheeler model [19] elsewhere and discuss this model's application in describing vacuum structure. We discuss this model in more detail in Section E. Ultrasonic waves with much longer wavelengths, λ, than the average mean free path, ℓ_e, of electrons are not scattered by the wave but ride up and down on the wave. At the lower temperature superconductivity state then, ℓ_e is much longer with the onset of the effect of Cooper pairs and there is a sizeable attenuation in ultrasonic waves in cold regions of astrophysical space. We will present the relationship of cold plasma interactions and fluid dynamic-like properties [20].

Resonance effects can be created by magnetic fields which vary in magnitude due to the periodic nature of the field of the electron, which is possibly generated by the vacuum lattice structure [5,20]. The topology of the Fermi surface governs the behavior of the electron in a magnetic field. The existence of the Fermi surface occurs because of the high density of electrons so that the Pauli exclusion principle dominates, wherein the electrons form a highly degenerate system in a quantum system for high density plasmons. The electron states are filled up to a certain level which is the Fermi energy. The Fermi surface is the constant energy surface of the Fermi energy, mapped out in momentum space [20]. Periodic forms exist within the surface due to the periodic nature of the lattice.

Again, we proceed from the usual definitions of the plasma frequency:

9. $$\omega_\rho = \left(4\pi\rho e^2 / m_e\right)^{1/2}$$

where ρ is the electron density and m_e is the electron mass. The Debye screening length is given as $\lambda_D = \left(4\pi\beta e^2 / \rho\right)^{-1/2}$, where β is the Boltzmann temperature defined as $1/KT$, K is the Boltzmann constant and T is the Kelvin temperature. The thermal deBroglie length is given as $\lambda = \hbar(\beta / 2m_e)^{-1/2}$. Quantum plasma properties dominate for $\delta > 1$, where $\delta = \hbar / \lambda_D$. We can write δ as

10. $$\delta = \frac{\hbar\beta}{\rho}\omega_\rho$$

In the collective description of our electron gas, the organized longitudinal oscillations cannot be sustained for wavelengths $\lambda < \lambda_D$ and occur only for coherent lengths $\lambda < \lambda_D$ which comprise the quantum picture. If we define the critical distance with a mean velocity $\bar{v}$ traveled in one oscillation, we have $\lambda_D \approx \bar{v} / \omega_\rho$. We can define the wavelength for collective behavior as $\lambda_c \approx c / \omega_\rho$ where $\bar{v} << c$ and where c is the velocity of light. That is, if the communication or information transfer velocity is large, then collective states will dominate.

We considered a simple example of an oscillatory imposed field $E = E_0 e^{-i(\underline{k}\cdot\underline{x} - \omega t)}$. If the frequency of oscillation of the field is high, then we must include the quantum mechanical properties of the medium, and when the photon energies are of the same order of magnitude as the electron rest energies, then the quantum properties of the radiation field must be included (see section VI). For the case of a high density plasma under low and high temperature conditions, we define a dimensionless quantity, r_s, which we will take to be small or of the order of the Debye screening length, divided by the Bohr radius. We define $r_s \equiv r_0 / a$ where r_0 is the interaction spacing of the order of λ_D and a is the Bohr radius. The volume per electron is $\frac{4}{3}\pi r_0^3$. Terms in $\frac{1}{r_s^2}$ are proportional to the electron density and r_s^2 is proportional to e^2, the electromagnetic coupling constant. If

11. $$r_s = \frac{e^2 m_e}{\hbar^2 \rho^{1/2}}$$

then the Fermi energy is given as

12. $$\varepsilon_\rho = \frac{3}{5}(9\pi / 4)^{2/3}\ 1/r_s^2$$

and the maximum electron momentum is given as

13. $$k = (9\pi / 4)^{1/3}\ \hbar / r_0$$

The Fermi energy levels are defined in terms of the vacuum state. The collective correlation energy is proportional to ε_F. The ground state $|\phi_0\rangle$ is the state of no electrons or holes and has the eigenvalue $\varepsilon_F = \sum_{k_1 > 1} \omega(k_i)$ for the momentum, k_i, of the i^{th} particle.

To consider both collective and single-particle motion, we separate the density of fluctuations of the plasma media into two parts: $\rho_k = \rho_{k\alpha} + \rho_{k\beta}$ which satisfies the oscillatory equation of motion $\ddot{\rho}_{k\alpha} + \omega^2 \rho_{k\alpha} = 0$ where $\rho_{k\alpha}$ represents the collective component associated with the oscillations, and the density $\rho_{k\beta}$ represents a collection of individual electrons surrounded by a cloud of charge which screens the field of electrons within the Debye length. This is our basic wave equation.

The ground state $|\phi_0\rangle$ then, in this model, is analogous to the vacuum state and any additional particles or holes with their polarization clouds are called quasiparticles. The screening aspect of the electron gas in terms of a renormalization of e^2, is automatically accounted for when collective behavior is considered. Coulomb divergences occur and thus the electron interaction must be renormalized. This approach is basic to the quantum plasma model.

The plasmon state is a resonant cooperative excitation of the density field which can decay by giving up its energy to various multiple excitations which are less correlated and coherent. This is the definition of the usual plasmon state. Also, collectivity of the plasmon state may be increased by the coupling of the excitations to the electron field and forming a state of greater coherence and resonance. This may be seen as soliton-like behavior. The plasmon and soliton states have no counterpart in a system of non-interacting particles where densities are extremely low, such as in interstellar space. The plasmon develops from a set of non-stationary density fluctuations. The plasmon excitations are acoustic modes which are longitudinal in their nature, and the soliton coherent states represent the mechanism of coherent growth through the process of nonlinear coupling which appears as plasma instabilities, but in reality are stabilities in terms of collective coherent behavior. However, since these states disrupt the plasma as observed in laboratory experiments, they are called instabilities.

Wave mode coupling is represented by pair creation or destruction of a plasma quantum. The processes of virtual and real pair production have an important effect on all plasma properties, such as electrical conductivity, dielectric constants and other electrical properties, as well as the spatial distribution of the gas itself. The electric parameters of the system couple directly to the external field and can thus be influenced by these fields. The spatial temporal plasma modes of excitation are also affected. External fields resonating with the internal plasma properties hence determine growth or decay of coherent modes. The key to the plasma coherent collective coupling process is expressed in the soliton formalism. These states can be maintained around specific conditions of black hole dynamics and give rise to certain structures in space such as supernovae. These astrophysical structures are maintained through the coupling of internal and external fields, both electromagnetic and gravitational. The coherent states of the plasma hence find a strong analogue to the exciton models in semiconductors and also the coherent excitonic modes in superconductivity, in which the Bardeen-Cooper-Schrieffer (BCS) formalism is given in terms of single particle and collective properties [21-24]. These states occur in interstellar space and near astrophysical systems where temperatures are near absolute zero.

The field-particle interaction is formulated in terms of the creation-destruction of particle-hole interactions which give rise to information and energy transfer between collective modes of the media arising out of single particle coherent excitations. These collective coherent plasmon modes occur because of the vacuum structure where a variety of energetic modes exist that access the electron-positron excitation modes of the Fermi sea model of the vacuum described herein. The degree of the effect of the polarized vacuum depends on the plasma density. Near a black hole, vacuum effects are large. Some of these excited states are called self-energy states. These collective states yield information about the structure of the vacuum itself (see section X).

C. Plasma Magnetohydrodynamics for the Vlasov-Maxwell-Poisson Semi-Classical Treatment

We proceed from Maxwell's equations for a system in an externally applied and internal field with the usual continuity equation for the Vlasov-Maxwell equation. Let us briefly outline the formalism so that we have a context for the quantization of the plasma and the description of the soliton plasma collective coherent states.

The electrodynamic processes of the plasma can be described by the use of the approximately collisionless Boltzmann or Vlasov equations that predict the damping of plasma oscillation modes. We will treat influences of collisions later. This damping process is the standard Landau damping where, in the quantum formalism, a plasmon or phonon or quantum of plasma oscillation decays, or is annihilated, into a one-particle final state or a collisionless, or nearly collisionless damped state. The condition for collisionless Landau damping is $\mathrm{T}_e >> \mathrm{T}_i$ where T_e is the electron temperature and T_i is the ion temperature. Needless to say, this picture does not carry the formalism of collective coherent processes, such as ones which include growth modes or coherent states of the plasma.

Let us start from the continuity or conservation of charge equation of the form

14. $$\frac{f\left(\underline{r}_i,\underline{v}_i t\right)}{\partial t}+\nabla\bullet\left(f\left(r_i,v_i,t\right)\right)=0$$

where $\left(f\left(r_i,v_i,t\right)\right)$ is the distribution function for the i^{th} particle or state. We can identify f_i with the density of series i [22].

We write Maxwell's equations in their usual form as

15. $$\frac{\partial \underline{E}}{\partial x_i}+J=c\nabla\times\underline{B} \qquad \nabla\bullet E=\rho$$
$$\frac{\partial \underline{B}}{\partial x_i}=-c\nabla\times\underline{E} \qquad \nabla\bullet B=0$$

where the momentum is $p = mv$ where v is the velocity, c is the velocity of light, and the current density is given as $\rho = \int f dv$ and the current as $J = \int \hat{v} f dv$. Also $\underline{E} = \nabla \phi$ and $\Delta \phi = \rho$ where the electric field is the gradient of the potential, ϕ. The constituent equations are

16. $$\underline{E} = \varepsilon_0 \left(\underline{E} + \underline{v} + \underline{B} \right) - \frac{\underline{v} \times \underline{B}}{c^2} \text{ and } \underline{B} = \mu_0 \left(\underline{B} - \underline{v}\underline{E} \right) + \frac{\underline{v} \times \underline{E}}{c^2}$$

In dealing with the collective modes of a two species plasma, n_i and n_j we use Poisson's equations

17. $$\nabla^2 \Phi = 4\pi\rho = 4\pi e \left(n_i - n_j \right)$$

where Φ is the potential and n_i and n_j are the number density of two species. In terms of thermo energy potential we can write

18. $$\nabla^2 \Phi = 4\pi e \left(e^{e\phi/KT} \right)$$

where e is the charge of an electron and the exponent is to the base e and K is the Boltzmann constant and T the temperature in degrees Kelvin. The spacing of particles in the plasma is given as λ_D, the Debye length between particles as $\lambda_D \cong 7\sqrt{T/h}$ for high temperature plasma, then the density n yields about 10^{10} to 10^{16} particles/cm^3. The interstellar plasma electron density is about $\rho \cong 1$ to 10 n_e / cm^3 at a temperature range of 10^2 to 10^4 $T_e{}^o K$. Stellar plasmas have densities of about 10^{15} n_e / cm^3, having a temperature range of about 10^7 to 10^9 $T_e{}^o K$. Note that the black hole density is many orders larger. This is the reason gravity and electromagnetism as well as the strong force, can come into balance.

The Poisson equation is given as

19. $$\nabla^2 \phi = -4\pi c^2 \int f\left(\underline{r}_i, \underline{v}_i, t \right) d^3 v dt$$

for velocity and temporal variations of $f\left(\underline{r}_i, \underline{v}_i, t\right)$. We use the form f_i to represent $f\left(\underline{r}_i, \underline{v}_i, t\right)$. The usual Lorentz force on particle i is given as

20. $$\underline{F} = f_i e \underline{E} + f_i \frac{v_i \times \underline{B}}{c}$$

where E and B are induced electric and magnetic fields in the plasma from external influences as well as internal plasma interactions.

Electromagnetic fields in the plasma medium can be described by the Vlasov-Maxwell-Poisson equations. Starting from $\nabla \times E = -1/c \partial B / \partial t$ and taking the curl of both sides we can then identify $\nabla \times B = 1/c \partial E / \partial t + J_e$ so that we have

21. $$\nabla \times \nabla \times \underline{E} = -\frac{1}{c} \frac{\partial^2 E}{\partial t^2} - \frac{1}{c} \frac{\partial J_e}{\partial \tau}$$

We assume that the time variation operator commutes with the del curl operator. Also we identify the current J_e with f_i as $J_e = \int \underline{v}^3 d\underline{v} f_i$ in velocity space. This gives us

22. $$\nabla \times \nabla \times \underline{E} = -\frac{1}{c} \frac{\partial^2 E}{\partial t} - \frac{1}{c} \int v d^3 v \frac{\partial J_e}{\partial \tau}$$

The fifth relevant equation for our electrodynamic problem is the momentum conservation equation, for effective mass m_i which is given as

23. $$m_i \left[\frac{\partial}{\partial t} \left(f_i \underline{v}_i \right) + \nabla \bullet f_i \underline{v}_i \underline{v}_i \right] + \nabla P_i = -e f_i E$$

for $E = \nabla\phi$, where in the simplest form f or f_i , $f_i \propto \sum_{k,\omega} f_0 e^{i(\underline{r}\bullet\underline{k}-\omega t)}$ for normalization constant f_0 , and ϕ is taken as an effective electric and/or magnetic potential of the form of $e^{-\beta e_1\phi_1}$ for $\beta = 1/KT$, the Boltzmann factor, and P_i is the pressure of species i , which we neglect for the first approximation. For our calculation for a specific geometry, we need to include the ∇P_i term for the plasma pressure. We will include this term in our calculation which involves the specific geometric configurations for natural and laboratory setups.

We use our magnetohydrodynamic (MHD) equations to determine acoustic resonant states in analogy to a classical soliton theory. We will also include quantum interactions of the electron-acoustic modes which also form coherent states. The role of the vacuum is also taken into account. The structure of the MHD system and also the usual hydrodynamics gives rise to simple coherent states with soliton-like properties. Quantum interactions enhance the stability of these states and the vacuum acts as an energy flux source. This source acts as a Prigogine system, giving rise to self-organizing properties of the media [23]. We will examine these issues in more detail in later sections. Note that we can identify the distribution function f_i with the particle number density n_i for particle species i .

D. Coherent Plasma States and Soliton Solutions to the MHD Equations

Using the MHD equations in the semi-classical approach given in the previous subsection, we will now demonstrate how the system of a fully ionized gas can form coherent resonances. These resonances are described as solitary waves. We will see that the solutions to the MHD equations do indeed give us soliton solutions. We examine two such solutions under different conditions, such as the velocity of propagation in the plasma, the electron and ion temperature, plasma frequencies, and external and internal field conditions. The treatment gives rise to a very good understanding of the formation of growth stability modes and collective states in the plasma and the whole issue of the soliton coherent state formulation and application.

In internal stellar and near event horizon conditions, temperatures can occur at hundreds of million degrees with plasma pressures of millions of gm/cm^2. Under these conditions, fusion can occur involving four main reactions with the release of exothermic energy where hydrogen is processed into helium [25,26]. Magnetic fields contain and control these conditions and capture highly energetic charged particles. Electrons can have energies of $10^5 eV$ and spiral around the lines of magnetic fields in gyroscopic helical paths. Gravity also acts as a plasma containment system in the sun and near black holes and other astrophysical systems. For example, under stellar conditions magnetic fields of a half-million to a million Gauss are present at over 100 atmospheres of pressure. Often these conditions can be controlled and/or affected by the dynamics of the large magnetic fields.

Under these conditions the plasma acts in a collective coherent manner in terms of nonlinear collective quantum states. These collective states involve nonlocal effects through the magnetic and gravitational fields and the vacuum state polarization. We can characterize these states in terms of solitary wave properties or soliton waves [27-29]. Turbulence near black holes in a dynamic state of formation can disrupt or enhance these modes and restabilize once conditions become more of a steady state.

We can calculate the speed and size of ion-acoustic solitons in the plasma. We consider a two-component nonisothermal plasma, $T_e >> T_i$, and low magnetic field, B , where

24. $$\rho \equiv 8\pi n T_e / B^2 << 1$$

where an external magnetic field is applied for an angle θ between B and the wave vector, $k << 1/\lambda_D$, where λ_D is the Debye length. We use the usual quasineutrality condition and include effects of strong nonlinearity. Charge separation effects become important when $\omega_{ci} \geq \omega_{pi}$ where ω_{ci} is the ion-cyclotron frequency, and ω_{pi} is the ion plasma frequency, thus the ions move in concurrent paths. This condition occurs near to or at plasma fusion conditions.

We can describe the plasma motion by the usual set of plasma equations; for the continuity equation,

25. $$\frac{\partial n}{\partial t} + \nabla \bullet (n\underline{v}) = 0$$

and

26. $$\frac{\partial \underline{v}}{\partial t}+\underline{v}\bullet\nabla\underline{v}=-\frac{e}{m_i}\nabla\phi+\underline{v}\times\underline{\omega}(g)_i$$

and the ion number density is given as $n=n_e=n_0e^{e\phi/KT_e}$ for ion mass, m_i, electron charge, e, velocity, $\underline{v}$, and ϕ is the electric potential. The ion gyrofrequency is given by $\underline{\omega}(g)_i=(eB_0/m_ic)z$ for $\underline{B}=B_0\hat{z}$ and n_0 is a constant.

We form a differential equation from the above three equations and find soliton solutions for certain conditions on relevant parameters. Solitons in the plasma density probably can be found for $v_p/v_s>\cos\theta$ where v_p is the speed of the plasma soliton and v_s is the ion-acoustic velocity. A few others have taken similar approaches [29,30].

The size of the ion-acoustic soliton in a magnetized plasma is characterized by

27. $$\rho_i=v_s/\omega(g)_i$$

where $\underline{\omega}(g)_i$ is the ion gyrofrequency. The ion acoustic speed is given as $v_s=(T_e/m_i)^{1/2}$. From the continuity equation for $\partial v/\partial t$, for the time variation of the number density we form

28. $$\eta\frac{v_x}{v_s}+\gamma\frac{\underline{v}}{v_s}=\frac{v_p}{v_s}\frac{((n/n_0)-1)}{n/n_0}$$

where $v=v_x\hat{x}+v_y\hat{y}+v_z\hat{z}$. Then we can write the plasma equation for $\partial\underline{v}/\partial t$ as three coupled equations:

29. $$\xi\frac{dv_x}{ds}=-\frac{\eta}{n/n_0}\frac{d(n/n_0)}{ds}+\frac{v_p}{v_s}v_y,\qquad \xi\frac{dv_y}{ds}=-\frac{v_p}{v_s}v_x,\qquad \xi\frac{dv_z}{ds}=-\frac{\gamma}{n/n_0}\frac{d(n/n_0)}{ds}$$

for

30. $$s=\frac{\eta\frac{x}{\rho_i}+\gamma\frac{z}{\rho_i}-\frac{v_p}{v_s}\omega(g)_i}{v_p/v_s}$$

and

31. $$\xi=\eta v_s+\gamma v_z-v_p/v_s\ .$$

These dimensionless forms of the equations allow us to write one differential equation for the above equations, as:

32. $$\frac{d}{ds}\left[\left(\frac{1}{n/n_0}-\frac{(v_p/v_s)^2}{(n/n_0)^2}\right)\frac{d(n/n_0)}{ds}\right]=(n/(n_0-1))(v_p/v_s)^2-\gamma^2(n/n_0).$$

Upon integration this equation can be written in the form of

33. $$\left(\frac{d(n/n_0)}{ds}\right)^2+\varphi(n/n_0)=0\ .$$

The $\varphi(n/n_0)$ term occupies the role of a classic particle "potential well". The form of $\varphi(n/n_0)$ is quite complex. Lee and Kan explore the form of φ and give analytic and numerical solutions [30]. Ion acoustic solitons exist for $\cos\theta<v_p/v_s<1$ and the normalized electric field, $E=0$ for the case where $n=n_0$, where

34. $$E=\frac{1}{(n/n_0)}\frac{d(n/n_0)}{ds}$$

Lee and Kan's approach yields

35. $$\varphi(n/n_0)=\frac{(n/n_0)^4}{((n/n_0)^2-v_p/v_s)^2}AB$$

for

36. $$A = 2\left(\frac{v_p}{v_s}\right)^2 \frac{n}{n_0}\gamma^2\left(1-\frac{n}{n_0}+\frac{n}{n_0}\ell n\frac{n}{n_0}\right)-2\left(\frac{v_p}{v_s}\right)^2\left(\frac{n}{n_0}\right)^2$$

and

37. $$B = \left(1-\frac{n}{n_0}+\ell n\frac{n}{n_0}\right)-\left(\left(\frac{v_p}{v_s}\right)^4+\gamma^2\left(\frac{n}{n_0}\right)^2\right)\left(\frac{n}{n_0}-1\right)^2+\eta^2\left(\left(\frac{v_p}{v_s}\right)^2-1\right)^2\left(\frac{n}{n_0}\right)^2$$

where η and γ are adjustable parameters. These parameters can be varied and some give coherent collective soliton states but are near unity. They can be affected by gravitational fields. Shukla and Yu have shown that finite amplitude ion-acoustic solitons can propagate at an angle to an external magnetic field in a plasma [31].

If we make some approximations, we can see more easily how we can obtain the Kosteweg-deVries equations [27]. Let us first take a one-dimensional space dependence only, for example,

38. $$\frac{\partial n}{\partial t}+\nabla\bullet\left(n\underline{v}\right)=0$$

becomes

39. $$\frac{\partial n}{\partial t}+\frac{\partial(nv)}{\partial x}=0$$

and for the continuity equation, we have

40. $$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{\partial U}{\partial x}$$

where we ignore the ion-gyrofrequency term of $\underline{\omega}(g)_i$ and U is a function of the electric potential, ϕ. For charge neutrality then we can write

41. $$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{1}{n/n_0}\frac{\partial(n/n_0)}{\partial x}$$

Charge neutrality puts a limit on the unlimited increase in an initial disturbance of the media which is damped by the presence of charge limits and the buildup of short-wave-length components of the disturbance.

Let us determine the associated wave solutions. We can define the Mach number $\Phi = v_0 / C_s$, which is the ratio of the pulse speed to the ion-acoustic speed. Note the similarity of Φ to our earlier ratio v_{ps}/v_s where v_{ps} is the speed of the plasma soliton and v_s is the ion-acoustic velocity. Essentially, $v_s = C_s$, and we can identify v_{ps} with v_0. Let us define a variable $\Theta \equiv x - \Phi t$.

For isolated pulse-like solitons we have the following boundary conditions

42. $$\lim|\Theta| = |x-\Phi t| \to \infty \text{ for } x \to 0.$$

Then $U = 0$, $v = 0$, $n/n_0 = 1$ and

43. $$\frac{\partial U}{\partial \Theta}=0,\ \frac{\partial(n/n_0)}{\partial\Theta}=0,\ \frac{\partial v}{\partial\Theta}=0.$$

Integrating the equations for $\partial n/\partial t$ and $\partial v/\partial t$ using the definition $\Theta = x - \Phi t$, we obtain

44. $$n/n_0 = \Phi/\Phi - v \text{ and } (\Phi - v)^2 = \left(\Phi^2 - 2U\right)$$

where we have used the limit of equation (42). We can define $U = e\phi / KT_e$. We will now assume the potential ϕ arises from electrostatic forces only or $E = -\partial\phi/\partial x$ in this case with no applied magnetic fields. For the Poisson equation,

45. $$\frac{\partial^2 U}{\partial x^2} = 4\pi e\left\{\frac{n}{n_0}e^{-e\phi/KT_e}\right\}.$$

We can write

46. $$\frac{\partial^2 U}{\partial x^2} \propto e^{-U} .$$

We can substitute equation (43) and (44) into the expressions for Φ and U and obtain

47. $$\tfrac{1}{2}\left(\frac{\partial U}{\partial \Theta}\right)^2 = \left[e^{-U} + \Phi\left(\Phi^2 - 2\right)^{1/2} - \left(\Phi^2 - 1\right) + C\right]$$

where C is a constant of integration, which we take to be zero.

This expression is similar to that previously obtained (equation (33)) for $\left(\frac{d(n/n_0)}{ds}\right)^2$ but is less complex because of our approximations and therefore the variable Θ is a simpler expression than that for s, etc. For Mach numbers slightly greater than unity, we obtain compressive solitary solutions which correspond to small amplitude waves with $U < 1$.

We can expand the above expression in orders of U and δ and retain leading order terms only. Then the above expression for $\tfrac{1}{2}\left(\frac{\partial U}{\partial \Theta}\right)^2$ becomes

48. $$\left(\frac{\partial U}{\partial \Theta}\right)^2 = \tfrac{2}{3} U^2 (3\delta - U)$$

which we now integrate

49. $$U = 3\delta \sinh^2\left[\left(\tfrac{1}{2}\delta\Theta\right)^{1/2} \Theta\right]$$

again for $\Theta \equiv x - \Phi t$. We find $U_{\max} = 3\delta\Phi$ for the maximum pulse and we used the definition $0 < \delta \equiv \Theta - 1 < 1$. We see that the solution U is indeed the form of a solitary wave! The half width of this wave is $\Gamma \propto (\delta)^{-1/2}$ [6].

This solution form is a more approximate form than our previous solution $\varphi(n/n_0)$, which indeed can also give soliton solutions [6,27,29]. We can now demonstrate that the soliton U satisfies the Korteweg-deVries equation. The variables n/n_0, U and v are series expanded and the lowest order terms are retained. Second order terms are defined as a set of variables rather than as x and t, and are used to define local disturbances. We return to the original set of equations for $\partial(n/n_0)/\partial t$ and $\partial n/\partial t$ and Poisson's equations. Then we can obtain

50. $$\frac{\partial(n/n_0)}{\partial \tau} + n\frac{\partial(n/n_0)}{\partial \chi} + \tfrac{1}{2}\frac{\partial^3(n/n_0)}{\partial \chi^3} = 0$$

where $\tau = fn(t)$ and $\chi = fn(x)$. The above equation is the Korteweg-deVries equation [30]. We can define $\chi = c^{1/2}(x - t)$ and $\tau = \varepsilon^{3/2} t$ for $\varepsilon = \delta = \Phi - 1$, in terms of the Mach numbers.

If dissipative processes occur, such as Landau damping, or magnetic fields are present, the above equation is modified. Equation (50) has solutions:

51. $$(n/n_0)(\chi - c\tau) = 3c\ \mathrm{sech}^2\left[\left(\tfrac{1}{2}c\right)^{1/2}(\chi - c\tau)\right]$$

and solitary wave solutions occur for what we term the "pseudo velocity", $c > 0$. This velocity of propagation of the soliton wave depends on the state of reference frame considered for the system, that is, fixed or rotating. If we proceed from the quantum field theoretic approach to MHD and then proceed to find soliton solutions, we will see that these solitons are solutions to the sine-Gordon equation rather than the Kortweg-deVries equation. As we have seen elsewhere, the solution form will be in terms of a sech^2 solution since this equation is a representation form of the classical Korteweg-deVries equation and quantum sine-Gordon formalism.

The excitation modes of the plasma are seen to arise from collective coherent states which couple to the energetic vacuum of the quantum state. In fact, the quantum picture gives us the mechanism and the description of the plasma media structure through which these collective modes arise. It is interesting to note that under certain critical conditions, the classical plasma physics also gives soliton acoustic mode solutions. The quantum picture gives us a more accurate representation of the conditions of the plasma in the vicinity of high gravitational fields near a black hole [32-34]. In the complete formalism, we treat the soliton wave as a mageto-acouston wave propagating in a strong magnetic field. The plasma–soliton coherent states have long-range coherent effects which are supported by the vacuum structure.

E. The Role of the Vacuum Energy in Physical Processes

A vast amount of energy is stored in the flux of the quantum vacuum. High energy processes such as high magnetic and gravitational fields near a black hole can activate and make observable the vacuum states. The vacuum energy has real physical observable consequences and its properties can be observed as having real physical effects [5,6]. These are extremely obvious in the vicinity of black holes.

Due to quantum uncertainty, seemingly "random" field fluctuations exist in the vacuum. Microscopic fields do not vanish and will arise as quantum fluctuations, although on a macroscopic scale electromagnetic field strengths average to zero these microfluctuations give rise to local energy variations and these quantum fluctuations arise from the energy-time Heisenberg Uncertainty Principle. This energy is powerful enough to create particles which live extremely short lives of about 10^{-20} seconds. Pair production from the vacuum does occur briefly and can be observed in the high field intensity near heavy nuclei. This charged pair represents a polarization of the vacuum and produces a minute but detectable shift in atomic spectra. The shift in the hydrogen levels is called the Lamb shift. A similar process of particle creation may occur in the vicinity of mini-black holes as well as astrophysical black holes [35-39].

The quantum vacuum fluctuation energy is given as $E = \sum_j 1/2\hbar\omega_j$ over a series of harmonic oscillators. Energy can be generated in the vacuum in a number of ways from external sources. This energy activates and excites the vacuum state so that the vacuum becomes observable through electron-positron pair production. The external energy, such as high magnetic field strengths and strong gravitational fields near superdense astrophysical bodies such as black holes or supernovae excite the plasma. It is through the energetic plasma states that the vacuum properties become apparent and observable. Under specific conditions with the correct available energy, coherent excitation modes appear and are like charged solitons in their properties. The precise form of the nonlinearities that give rise to the soliton structure can be formulated in terms of the complexification of the set of relevant equations such as Maxwell's equations [38] or the Schrödinger equation [39]. The imaginary terms in these equations can be utilized to describe soliton coherent states. In reference [39], the effects of the actual coherent states and its application to the vacuum can be made. Boyer details the field theoretic approach to describe vacuum processes [40]. Also the experimental test of the existence of zero-point fluctuations is detailed, such as the Lamb shift, Casimir effect, and possible effects on long-range electromagnetic fields [41,42].

Very energetic processes cohere the vacuum and create real physical effects. The question is if one can enhance this coherence and utilize it to optimize macroscopically observable "energy shifted" states. It is clear that the vacuum plays a role in physically realized states. The question then becomes, can we enhance the role of the vacuum to form interesting and utilizable processes in materials with coherent excitations that would be observed as apparent ambient superconducting states [21]. Let us briefly give another example of the role of the vacuum in physical theory, for example in chromoelectrodynamics theory, where we represent the properties of the vacuum as a form of soliton called an instanton which is a time-dependent entity rather than space-dependent like a soliton. We treat the relationship between quantum electrodynamics, QED and quantum chromodynamics in separate papers [4,43-45]. In the chromodynamics theory of elementary particle physics, the charged particles are quarks and their fractional charge is called the "color" quantum number. The field quanta by which the quarks interact are called gluons. Instantons arise out of the solutions that describe the forces in the chromodynamic field. They are properties of the vacuum. Since the vacuum is defined as "zero energy" they are essentially "pseudo-particles". But instantons have a real physical effect; in their presence the gluons "feel" forces arising from the non-empty vacuum [4,44,45]. Solitons are coherent in space and instantons are coherent in time. In work in progress, we address the strong force and color force as consequences of a quantum gravity where a torque term and Coriolis effects are incorporated in the Hamiltonian of a nonlinear Schrödinger equation.

The work of Lindquist and Wheeler fits well with our model of the vacuum structure. Briefly stated, this work involves the Schwarzschild cell method which considers the dynamics of a lattice universe as a consequence of Einstein's field equations. These equations are fulfilled everywhere except at the interface between "zones of influence" [19]. The lattice universe by the Schwarzschild method yields an interesting picture of the vacuum. It has been noted that the elementary potential form of $1/r$ exists for a point charge in the Coulomb interaction. Also we note that the Schwarzschild metric contains an analogous $1/r$ potential for the ten Einstein gravitational metric potentials. Here $Q = S = 0$, which is only an approximation to our balance equation because we consider $Q \neq 0$ and $S \neq 0$ and $c \neq 0$. Then

52. $$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

We now see a method of relating the Coulomb and gravitational potentials. Inside each domain of action of the potentials, we replace the actual gravitational potentials by the Schwarzschild expression. This treatment, uses the electronic wave functions which are derived from crystal lattice work and is extremely fundamental to our work [20,45].

Note that the third term in the Schwarzschild derivative is proportional to the Newtonian gravitational term GM / r^2. So that the cells do not nullify each other, the equations of motion at the center of the cell are under a dynamic condition as is the cell boundary. The Wigner and Seitz method is used in analyzing the electronic wave functions in crystal lattices [46]. The Lindquist and Wheeler method depends on the mass of the singularities in an asymptotically flat space. Symmetry arguments from lattice structure approaches require the decomposition of all curved space into Schwarzschild cells. In the four-dimensional Euclidean space, the authors mark out vertices of regular geometric figures of the lattice universe. Particles can specify the vertices, where the nearest neighbors for $n = 5$, 8, 16, 24, 120, 600 correspond to the tetrahedron, cube, tetrahedron, octahedron, dodecahedron and tetrahedron again respectively (see section 10).

We can compare this approach to our group theory and GUT theory and crystallographic point group theory [47]. What we observe in the Lindquist and Wheeler approach is a method of directly relating the electromagnetic field and gravitational field at the level of fundamental geometric structure. We can construe that such a form not only governs the vacuum structure but uniquely relates electric and gravitational fields. The lattice universe space is closed but not by everywhere uniform curvature as in the Friedmann universe [19]

This is the point of our discourse and leads to the concept of a structured vacuum, which manifest stellar, galactic, and extra galactic dynamically gravitationally collapsing black hole systems. Here we have a new methodology for unification of fields and geometric scales. The shape of a typical cell is like a deformed cube in the case of an eight particle lattice universe. Three cells meet at an edge rather than the four in Euclidean geometry.

F. The Quantum Formalism and Perturbation Analysis in Plasma Physics

We can use the quantum formalism to calculate the density of the plasma undergoing collective oscillations under an externally applied field and under its own internal collective states. We quantize the classic wave equation for particle oscillations in terms of the second quantized formalism. We will examine in some detail the plasma electron excitation of the electron-hole pairs of the Fermi sea vacuum states. For the purposes of the present calculation, we will treat the ions as fixed [48].

We can formulate the plasma collective states in terms of perturbation or interaction propagator. The perturbation theory can be used to treat a many-body particle interaction forming a collective plasmon (or phonon) state. We can form a perturbation series from our Schrödinger/Hamiltonian equation $HU = EU$ [49,50]. This perturbation series is expanded in terms of a propagator where we use projection operators to project out observed states. The perturbation series can be written as a series expression in term of Feynman integrals. In this way we can picture the role of the energy of the vacuum in creating perturbations in the plasma giving rise to collective plasmon states [5]. This method has been used with a great deal of success both in superconductivity theory and solid state physics [51,52].

In a potential field we can write a more general expression for the wave function:

53. $$U(r,t) \propto e^{\frac{1}{\hbar}\left(\underline{k} \bullet \underline{r} - \omega(\underline{k})t\right)} \chi_\sigma$$

where χ_σ is the internal spin wave function with the usual eigenstates for electrons being

54. $$\sigma = \pm 1/2 \text{, and } \omega(k) = -V + \frac{k^2\hbar^2}{2m}$$

where V is the potential experienced by the electron. The wave function is then expanded in terms of the creation and destruction operators, a, a^+. The ground state is equivalent to the vacuum state in quantum electrodynamics in a phenomenological approach to field theory, where the state of the form $|\phi_0\rangle$ is the noninteracting state of the system where there is no excitation of electrons and holes above the fermi surface. The state $|\phi_0\rangle$ is not identical to the empty vacuum $|\phi\rangle$ because of the so-called "passive particle" states which constitute the full vacuum of the Fermi sea model.

Using the propagator techniques of reference [5,6], we can write the density of plasma states from our perturbation formalism. We have a charge density operator, $\rho(\underline{r},t)$, which we can write in terms of our second quantized electron field operator. The Hamiltonian in terms of ρ can be written as the Coulomb potential V,

55. $$H = \frac{2\pi\hbar^2 e}{V}\left\{\sum_j \rho_j \rho_{-j}\frac{1}{q^2} - N\sum_j \frac{1}{q^2}\right\}$$

where

56. $$\rho_j(t) = \int d^3\underline{r}e^{i\underline{q}\bullet\underline{r}\rho(\underline{r},t)}$$

and $\rho(\underline{r},t)$, the density operator, is expressed as

57. $$\rho(\underline{r},t) = \sum_\sigma \varphi_\sigma(\underline{r},t)\varphi_\sigma(\underline{r},t)$$

for field operators $\varphi_\sigma(\underline{r},t)$ expressed in terms of the operators a^+ and a, and $\rho_j = \sum_{k,\sigma} a^+_{k,\sigma}$, which is the propagator for density fluctuations in the electron-hole field. It is found that correlations form in the density fluctuations. The propagator is interpreted as the amplitude for the propagation of electron-hole or electron-positron pairs.

The singularities in the propagator function are of interest and represent the correlated oscillations of the electron density field. These singularities represent the phonon excitations that occur in the density field which is analytically continuous in the momentum plane, k. The singularities arise as a continuous distribution of poles which correspond to the possible energies of pair states. In references [5] and [6] one of us (Rauscher) has demonstrated the manner in which the density fluctuations can occur in the medium due to electron scattering. The resulting polarization or induced charge can then, in turn, affect one of the electrons by means of the Coulomb interaction.

The virtual pairs are produced from the excitation of the vacuum and are then equivalent to the density fluctuations which we have calculated. The important effects of the electron interactions on the properties of the electrons in the plasma arise from the modifying influence of the induced density fluctuations, and explain the manner in which plasma collective behavior arises. All the plasma properties are modified by the virtual state vacuum polarization. In reference [5] one of us (Rauscher) calculates the modification of the dielectric constant. Conductivity and other plasma properties are also affected by the existence of properties of the vacuum.

By including the appropriate series of Feynman graphs which represent the electron excitation of vacuum pair production, we find an adequate calculation of the observed plasma dielectric constant. The leading order term is the classical value and higher order terms give additional contributions of about 15% to match the observed values which demonstrates that quantum effects and vacuum state polarization have real physical properties. With the quantum approach, we can calculate the properties of the plasma more accurately. We can thus understand better the manner in which collective plasmon or phonon states arise as electron activation of electron-hole pairs from the Fermi sea vacuum state and what such formalism says about the properties and structure of the vacuum.

We have examined a model in which we treat the interaction of these collective phonon modes, from the electron pair creation, to the electrons of the plasma and treat this state as a soliton state which maintains its identity over nonlocal space and time. As in our earlier treatment, we consider fixed positive ion states, but as we see, these states, such as in lattice structures, can also contribute to phonon vibrational states, for example in the Lindquist-Wheeler model [19]. Elsewhere, we have introduced the formalism for effect of coherent energetic plasmon magnetic acouston states on the vacuum [5,6].

G. Detailed Structure of the Vacuum State

If we proceed from the empty null vacuum state $|\phi_0\rangle$ we can express the probability of this empty state under no action from the vacuum as $\langle\phi_0|\phi_0\rangle$. If, however, a process is occurring in the vacuum, we can express this as some operation operating on the vacuum as $P_{op}|\phi_0\rangle$. If coherent energy and entropy is supplied to the vacuum, and the vacuum has a structure, then we can denote this condition as $P_{op}|\phi_n\rangle$ where n is a term in some series from 0 (the ground state) to $n=N$, where N can be large or go to ∞. Effects on vacuum states from external sources can produce a variety of properties in an energized medium, including polarization, changes in conductivity, and other electromagnetic phenomenon.

We can associate each geometric, crystal form with a specific group for that form. A group is a collection of objects, such as mathematical symbols that are related by a set of algebraic operations. The generators of the group for the set of elements of the algebra can be commutative, such as Abelian $[x_i, x_j]=0$, or non-Abelian as $[x_i, x_j]\neq 0$. In a schematic representation we can state that the group $=e^{a\lg}$ where the term alg stands for the algebra or actually, the generators of the group which are the elements of the algebra. For a Lie group, the generators are infinitesimal generators and form a Lie algebra.

For an Abelian group the elements of a commutative relation are expressed as exponents of log base e. The group is a sum of matrix representations. We represent the generalized commutation as $\Omega_n(A,B)$ where A has elements x_i and B has elements x_j. We have an expansion

58. $$e^{tA}=\sum_{n=0}^{\infty}\frac{t_n}{n!}A^n$$

where e^{tA} represents a unitary transformation and $n!$ is the product $1\times2\times3..n$.

Starting with the square matrix representation $A=(A_{nm})$ for $n=m$. Then $\Omega_n(A,B)$ represents the commutation relation and the zeroth order $n=0$ is given as $\Omega_0(A,B)=B$ and the first order $\Omega_1(A,B)=[A,B]$ and $\Omega_2(A,B)=[A,[A,B]]$ etc. In general, then, $\Omega_{n+1}(A,B)=[A,\Omega_n(A,B)]$ for higher order commutation relations. For a Lie algebra $[A,B]=AB-BA\neq 0$.

A formal series of the group representations can be written as

59. $$e^{tA}Be^{-tA}=\sum_{n=0}^{A}\frac{t^n}{n!}\Omega_n(A,B)$$

which can be a unitary transformation. Now returning to e^{tA}, which can represent a unitary transformation, we have the above expression where A and B are square matrices and $\Omega_n(A,\Omega_m(A,B))=\Omega_n+m(A,B)$ as a formal power series. We can say that x_i are the elements of $A, x_i\supset A$, and x_j are the elements of $B, x_j\supset B$. In general terms $e^x=1+x+x^2+x^3....x^n$.

In order to describe the energetic properties of the vacuum, we construct an energy Hamiltonian wave equation which describes the wave equations for interstellar, stellar, galactic plasma and plasmas surrounding black holes. Let us proceed from the classical wave equation

60. $$\frac{\partial^2 U}{\partial x^2}=\frac{1}{c_0^2}\frac{\partial^2 U}{\partial t^2}$$

where c_0 is the characteristic velocity of the wave with amplitude $U(x,t)$. We utilize the simplified two dimensional form. The solution to this equation unveils a "left going wave" $U = e^{-i(\underline{k}\cdot\underline{x}-\omega t)}$ and a right going wave $U = e^{i(\underline{k}\cdot\underline{x}-\omega t)}$ which sets up a standing wave in the plasma medium.

We can write the classical equation of motion for such a system in terms of the energy Hamiltonian, H. The momentum, p, and spatial dimension, x, also termed for the temporal dimension, t, yields the plasma space relations. We have

61. $$\dot{p} = \frac{\partial p}{\partial t} = -\frac{\partial H}{\partial q} \text{ and } \dot{q} = \frac{\partial q}{\partial t} = -\frac{\partial H}{\partial p}.$$

The equation of motion in terms of q or x is $\ddot{q} + \lambda q = 0$ where

62. $$H = \frac{p^2}{2m} + \lambda q^2$$

where m and λ are constants dependent on the particles in the plasma undergoing motion where m is a mass like variable and λ acts like a potential.

We then express paired p, q, which are canonically conjugate variables expressed in terms of the wave amplitude, U, where U^* is the complex conjugate of U. Then

63. $$p = \frac{i}{\sqrt{2}}\left(U - U^*\right) \text{ and } q = \frac{i}{\sqrt{2}}\left(U + U^*\right).$$

For non-abelian operators, we have the quantum condition $[p,q] = -i\hbar$ where $\hbar$ is Planck's constant and for abelian algebras $[p,q] = 0$ for the classical conditions.

We can construct creation and destruction operators from the vacuum state. Then

64. $$a = \frac{1}{\sqrt{2\hbar}}(q + ip) \text{ and } a^+ = \frac{1}{\sqrt{2\hbar}}(q - ip)$$

in the Fermi-Dirac statistics (half integral spin) apply at the micro particle level for the interaction Hamiltonian $H' = \sum_i \sum_j A_{ij}\left(a_i a^+ + a_i^+ a_j\right)$. The a's are the particle creation operators and the a^+'s are the destruction operators where a^+ is the complex conjugate of a. When energy enters the vacuum from, for example, γ rays impinging on a target, it will produce election-positron states. Positrons are created and electrons destroyed or absorbed into the vacuum where the positrons arise. The operators a^+ and a can create or annihilate a pair of energy quanta of the plasmon or phonon states. The energy Hamiltonian for the system is $H = \hbar\omega\left(a^+ a\right)$ where $HU = EU$. In the many Fermion spin ½ problem, we can expand the vacuum energy in a series of terms in analogy to the series of generator terms that make up the group representation of the structured vacuum. Thus we have the ground state Hamiltonian as $H_0|\phi_0\rangle = E_0|\phi_0\rangle$ and the interacting perturbed or perturbed Hamiltonian as $H_1|\varphi_0\rangle = E_0|\varphi_0\rangle$ [5].

In Perturbation Theory, $H = H_0 + H_1$, then we can write $H_0|\phi_0\rangle = \omega_0|\phi_0\rangle$ for the unperturbed, non-interacting state, and $H_1|\varphi_0\rangle = E_0|\varphi_0\rangle$ for the perturbed, interacting state. We obtain the form for the excited states as

65. $$|\varphi_0\rangle = |\phi_0\rangle + \frac{\Lambda}{E_0 - H_0} H_1 \varphi_0$$

where $\Lambda = 1 - |\phi_0\rangle\langle\phi_0|$ which is the projection operator which projects the state $|\phi_0\rangle$ and $\langle\phi_0|\phi_0\rangle = 1$ and $\langle\phi_0|\varphi_0\rangle = 1$ for the normalization conditions. The term $\frac{1}{E_0 - H_0}$ acts as a propagator of the excited states. The ground state, $|\phi_0\rangle$ equivalent to vacuum state, $|0\rangle$ and the ground state energy is given in terms of higher energy states and,

66. $$E_0 = \varepsilon_0 + \langle\phi_0|H_1|\varphi_0\rangle$$

which upon interaction becomes a series expansion. We expand from

67. $$(H - E_0)|\phi_0\rangle = (H_0 + H_1 - E_0)|\phi_0\rangle = 0$$

and upon interaction, we obtain

68. $$E_0 = \varepsilon_0 + \langle\phi_0|H_1|\phi_0\rangle + \langle\phi_0|H_1 \frac{\Lambda}{E_0 - H_0} H_1|\phi_0\rangle + ...$$

The representation of the terms in the perturbation series are given by Feynman graphs. The state $|\phi_0\rangle$ occupies an analogous role in this theory for the many-body plasma states where $|0\rangle$ is the unperturbed vacuum state as is done in field theory. Field theory technique can be well adapted to our vacuum plasma model. What we can demonstrate in this approach is that we can form a mathematical relationship between a geometric structured vacuum and the usual picture of the Fermi-Dirac vacuum. Hence, our model brings us beyond the standard vacuum model to that of a structured vacuum which is congruent with observed structured forms in astrophysical and cosmological phenomena. This picture also effectively describes the intense energy interaction in the region of event horizons of stellar, astrophysical and cosmological black holes and their surrounding plasma media. These energetic processes determine the form, shape and structure of the observed astrophysical and cosmological structures. Hence the microgeometric forms of the vacuum drive these macroscopic forms.

We have detailed elsewhere the particular associated group and its group generators with specific geometric forms. This will relate the driving forces and energies of the micro vacuum structure to observed macroscopic cosmological events. Note that these derivations can lead to some detailed calculations of design parameters for laboratory experiments.

V. RELATIVISTIC CONDITIONS ON THE BALANCE EQUATIONS AND THE ENERGY DENSITY OF THE PLASMA

In this section we detail the balance of gravitational forces with the surrounding electrodynamic plasma media. Relativistic invariance conditions apply. In the dense plasma media, standing coherent wave modes are set up between the ergosphere and the outer regions of the plasma field where the density of the plasma drops off to a collisionless media. We denote r_s the Schwarzschild radius (the approximation for the region of the inner radius) and ℓ the radial distance to the outer regions where the plasma density drops below the energy E_n. In this case, the plasma density becomes lower than the effective density for plasma-vacuum interactions. In this section we examine the propagation of electromagnetic waves in plasma in a region of a gravitational field near an astrophysical body.

The wave dynamics for the balance equation of plasma matter near the ergosphere can act as a coherent standing-wave pattern. We can derive the equation of this state in terms of a coherent soliton wave with low dispersive loss. Standing wave patterns drift towards areas of low wave velocity. Two opposing forces occupy a role in the dynamics of this wave pattern. The gravitational force acts proportionately to the gradient of the wave velocity squared, ∇v^2, and opposing this force, in the action of the plasma media, is the force of inertia, which is proportional to the mutual acceleration of the wave pattern form and of the plasma media. The constant of proportionality for both forces equals the total vibrational or oscillatory energy of the soliton wave divided by its velocity squared or $\frac{E_s}{v_s^2}$. It appears that Lorentz symmetry of the medium may hold. For solitary waves or solitons the wave amplitude U is proportional to its velocity squared v^2 and under the influence of the acceleration of gravity g then $U = v^2 / g$.

The unconstrained motion of standing electromagnetic wave modes coupled to the vacuum obey the gross mode or collective behavior of collective particle states moving along geodesic lines of gravitational forces. This requires that the properties of the vacuum be considered. Boundary conditions are necessary to confine the standing wave pattern which balances the gravitational force and the electromagnetic energized plasma media. The black hole event horizon is the dividing boundary between the strong gravitational attractive field and the collective coherent oscillatory field of the plasma medium. The energy of the media results from the electromagnetic, thermodynamic, and highly ionized vacuum coherent states.

Let us first consider the one dimensional d'Alembertian wave equation of the collective states of the plasma. We take the variational derivative of the Lagrangrian as

69. $$L = \frac{1}{2\mu}\left(\frac{\partial U}{\partial t}\right)^2 - \frac{1}{2\varepsilon}\left(\frac{\partial U}{\partial x}\right)^2$$

the wave distribution is given as U , and μ and ε are electromagnetic parameters so that $v = \sqrt{\varepsilon / \mu}$ where v is the wave velocity, and the wave impedance is $z = \sqrt{\varepsilon\mu}$. The velocity, v , can be taken as the velocity of light. The Galilean transformation of time, $t \to t$ or $x \to x + vt$ will give us the new Langrangian function from the above equation as

70. $$L = \frac{1}{\partial\mu}\left(\frac{\partial U}{\partial t}\right)^2 - \frac{1}{2}\left(\varepsilon - \mu v^2\right)\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)\left(\frac{\partial U}{\partial t}\right).$$

We can write a wave equation for this Lagrangian by taking the variational derivative of the above equation. The states of the plasma medium and its motion obey the following conditions:

71. $$\frac{\partial v}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial t} = 0$$

for no accelerative elasticity in the media

72. $$\frac{\partial \mu}{\partial t} = 0 \text{ and } \frac{\partial \varepsilon}{\partial t} = 0$$

for the time independent rest frame and

73. $$\frac{\partial \mu}{\partial x} = 0 \text{ and } \frac{\partial \varepsilon}{\partial x} = 0$$

for the spatial inhomogeneity in the rest frame.

Then we construct the wave generalized equation from our Lagrangian as

74. $$\frac{\partial^2 U}{\partial t^2} - \left(c^2 - v^2\right)\frac{\partial^2 U}{\partial x^2} + 2v\frac{\partial^2 U}{\partial x \partial t} + \frac{1}{\mu}\frac{\partial \varepsilon}{\partial x} + v^2 \frac{\partial \ell n \varepsilon}{\partial x}\frac{\partial U}{\partial x} = 0$$

which differs from the standard wave equation by the additional terms $\frac{\partial^2 U}{\partial x \partial t}$, $\frac{\partial \varepsilon}{\partial x}$, and $\frac{\partial \ell n \varepsilon}{\partial x}$ for electromagnetic wave propagation in a gravitational field where n is an integer.

Under the conditions of local Lorentz symmetry, without applying the normally accompanying Poincare homogeneity we let $v = 0$ and

75. $$\frac{\partial v}{\partial t} = a \neq 0$$

where the acceleration, a , is a Lorentz observable. For vacuum conditions $z = \sqrt{\mu\varepsilon}$. The permittivity is ε and the permeability is μ . The electric displacement field $D = \kappa\varepsilon_0 E$ where κ is the dielectric constant. Note that z became a more complex term when we use the phase invariant condition in a plasma. In fact $z \neq \sqrt{\mu\varepsilon}$ and $c \neq 1/\sqrt{\mu\varepsilon}$ when we consider more complex dispersion relationships as a function of collective frequency ω and wave number k so that $z = \sqrt{\mu\varepsilon}$ becomes a first order term where $c = \omega k$. In the general case $c \neq \omega k$ but is expressed as a non trivial function of ω and K . In terms of the dielectic constant, κ , we can express μ and ε

in matter in terms of their vacuum counterparts as $\varepsilon = \kappa\varepsilon_0$ and $\mu = \kappa\mu_0$ for $(\mu/\varepsilon)^{1/2} = (\mu_0/t_0)$ in order to satisfy Eötvös type experiments [50].

In the simplest case the plasma oscillations satisfy an equation of motion of the collective state density of $\ddot{\rho}_{k\beta} + \omega^2\rho_{k\beta} = 0$ where $\rho_{k\beta}$ represents the collective component and $\rho_{k\alpha}$ represents the individual particle component in the plasma. The wave amplitude, U , is analogous to ρ_k . For the approximation of $z = \sqrt{\mu\varepsilon}$ then

76. $$\frac{d\mu}{\mu} + \frac{d\varepsilon}{\varepsilon} = 0 .$$

This condition implies that the inhomogeneity of the vacuum does not lead to reflection and scattering and most of the energy is radiative. This is a fair approximation except that scattering may need to be considered in a super dense plasma. However, scattering effects can be taken into account by the formation of collective coherent states in the plasma.

Using the condition for $v = 0$ and $a \neq 0$ and $z = \sqrt{\mu\varepsilon}$ then our wave equation (74) becomes

77. $$\frac{\partial^2 U}{\partial t^2} - c^2\frac{\partial^2 U}{\partial x^2} + \left(a - \frac{1}{2}\right)\frac{\partial c^2}{\partial x}\frac{\partial U}{\partial x} = 0 .$$

We thus have the standing wave equation between the comoving boundaries of $x = r_s$, which we take as the zero boundary $r_s = 0$ and $x = \ell$ in the moving frame of reference. We can consider the simplifying assumption, $a =$ constant and

78. $$\frac{\ell}{c^2}\frac{\partial c^2}{\partial x} < c < 1 .$$

Then the normal mode solutions that vanish at the boundary of the event horizon are:

79. $$U_n(x,t) = A_n e^{\zeta} \sin C_n \times \sin \omega_n t$$

where

80. $$\zeta = \left(\alpha / 2v^2\right)x = \frac{\alpha}{2v^2}x$$

and

81. $$C_n = n\pi / e$$

and

82. $$\omega_n = n\pi v / \ell$$

where

83. $$v = \ell / t = \ell\omega .$$

The time average of the energy density that is energy per unit length, is given or, using the above solution for U_n , then

84. $$\langle E_n \rangle = \frac{1}{2}\omega_n^2 A_n^2 e^{2\zeta}$$

in which higher order contributions of the order of v^{-4} are neglected. This is a good approximation for our conditions.

The experimental dependence of $\langle E_n \rangle$ on x produces an asymmetry in the radiation pressure on the fixed boundaries which result in a net force F between the media and the event horizon boundary as

85. $$F = \langle E_n(\ell) \rangle - \langle E_n(0) \rangle$$

Utilizing the expression for $U_n(x,t)$ and substituting it into the above equation for force, F , we have

86. $$F = \frac{1}{2}\omega_n^2 A_n^2 \left(e^{2\zeta} - 1\right)$$

In the case where $\alpha\ell << v^2$ and using

87. $$\alpha = a - \frac{1}{2}\frac{\partial v^2}{\partial x^2}$$

we obtain

88. $$F = \frac{E}{v^2}\left(a - \frac{1}{2}\frac{\partial v^2}{\partial x^2}\right)$$

where we take $E \equiv \langle E_n \rangle \ell$ in the first approximation of the total vibrational energy which is contained in the standing wave pattern between the event horizon boundary and the outer boundary where the plasma density is negligible.

The normal mode energy are additions so that $F = \langle E_n(\ell)\rangle - \langle E_n(0)\rangle$ holds for an arbitrary wave pattern satisfying the boundary conditions. The distance ℓ defines the region around the black hole where the plasma is energetic enough to polarize the vacuum. Note that we use the inner boundary condition as $x = r_s = 0$ at the event horizon.

We interpret the relation for the force in equation (88) as the condition that the acceleration of a standing wave pattern is associated with an inertial force which is determined by the equivalent mass $m = {}^{2E}\!/_{v^2}$ or ${}^{E}\!/_{c^2}$ where $v = c$, the velocity of light. The standing wave pattern can shift position towards a place where ${}^{\partial v^2}\!/_{\partial x^2} = 0$ and seek a position where v is minimal.

For electromagnetic waves, where the vacuum states are significant such as in a plasma, the geodesic line concept holds for the spacetime relativistic theory of gravity where

89. $$\alpha = a - \frac{1}{2}\frac{\partial v^2}{\partial x^2}$$

where α is equivalent to the Newtonian approximation of the geodesic line equation and a is the acceleration. This will hold in the plasma media where ℓ is outside of the event horizon r_s and not where $\ell = r_s$ where r_s is approximately the Schwarzchild radius. Thus the Newtonian gravitational potential is a weak perturbation of the order of v^2.

Since our result for electromagnetic plasma waves in a vacuum obeys the geodesic line hypothesis in the spacetime theory of gravitation (see equation (89)) relativistic conditions apply. Hence the gravitational potential acts as a weak perturbation of v^2 when $v^2 \sim c^2$ in the fast wave approximation [43]. In fact, the soliton wave moves more slowly than the velocity of light, c. Our wave equation (77) applies for a more arbitrary media described in the more generalized wave equation in which $v \neq 0$ and $a \neq 0$. Under the conditions where $v << c$ which is our plasma case. These conditions work well in our nondispersive media or in fact where dispersive losses are balanced by the nonlinear terms to achieve the soliton conditions. However, the full treatment requires more general dispersion relations and more detailed consideration of the nonlinearties of the plasma in which we relate the collective frequency, ω, and wave number, k. Thus the impedance $z \neq \sqrt{\mu\varepsilon} \neq$ constant. The condition

90. $$\frac{\partial\mu}{\mu} + \frac{\partial\varepsilon}{\varepsilon} = 0$$

is essential to obtain the gravitational potential gradient

91. $$\frac{1}{2}\frac{\partial v^2}{\partial \varepsilon}$$

to include gravitational effects on the electromagnetic field.

At the outer regions of the plasma "matter wave" surrounding a black hole for the boundary at radius ℓ about the black hole the plasma gas can exchange $r > \ell$ with $r < \ell$ but the standing wave solutions require that the waves act between r_s and ℓ but the plasma media is not confined to this constraint.

This approach involves some approximations which we may need to relax because of the nature of the plasma near the region of r_s. We have even developed a coherent description of electromagnetic wave momentum in analogy to particle momentum or "matter waves" in a gravitational field. This approach retains the symmetry of the stress-energy on the momentum-energy tensor of Einstein's field equations. We now need to consider these electromagnetic "matter waves" or coherent wave structure where $z \neq \sqrt{\mu\varepsilon}$ and where particle collections in a dense plasma apply.

For our amended field equations with the torque term and the resulting Coriolis effects, we are able to accommodate these conditions. Due to the Coriolis effect and as a result of the plasma dynamics, we can observe that torque is the driving force of the plasma field effects and hence the source term of the balance equation. The collective coherent states propagate information and their effects throughout the plasma medium. It is through these coherent states that the effect of torque is transmitted throughout the medium surrounding the black hole and is observable as the dynamics we observe in supernovae and other astrophysical objects.

VI. RELEVANT THERMODYNAMIC PROCESSES IN ASTROPHYSICAL SYSTEMS AND THE BALANCE EQUATION

Energy is transferred by conduction, radiation, and convection. Interspatial energy transmission is dominated by radiative processes. Conduction occurs on contiguous surfaces in stellar interiors and convection occurs on the surface and through the stellar systems. Individual radiative processes do occupy a role in forming collective thermodynamic emission and absorption processes. Planck's spectral radiation law from $E = \hbar\nu$ does apply but is dominated by much more complex dynamic collective processes. Debye's approach treats a system as a continuum rather than a system at the individual particle levels, which is much more applicable to our extremely high temperatures case. For example, in the strong gravitational collapsing field in the vicinity of a black hole, activation of energetic processes in the surrounding gases can occur and form plasmas.

Debye's so-termed T^3 law, where T is the absolute temperature in Kelvin, was first developed for solid crystalline or amorphous systems and can apply to near zero degree systems. Such systems can be made analogous to interstellar and intergalactic dark matter (nonluminous matter regions), galactic halos, and perhaps, can be extended to the surface of neutron stars and the vacuum of the quantum domain. Proceeding from the energy content, E, and heat constant for the materials under consideration as well as the gas constant, R from the energy constant RT / N, we have $E = aT^4$ where

92. $$a = \frac{3}{5}\pi^4 \frac{R}{\beta v^3}$$

and thus depends on the radiation frequency emitted and also

93. $$\beta = \frac{1}{KT}\,.$$

Very high pressure systems may require the Polanyi [53] treatment at finite temperatures or

94. $$\frac{dA}{dT} = \frac{dU}{dT} = 0$$

as an approximation. This expression is derived from the Nernst [54] heat theorem based on thermoelectric materials. The energy associated is U, and the effect is the work, W, at temperature, T. Note that the entropy is

95. $$S = \frac{dW}{dT}\,.$$

The radiative emissions processes make possible the detection of black hole dynamics from earth based observatories. The exact mechanisms of black hole radiation are not well understood, yet they are critical to our understanding of the fundamental nature of stellar and galactic black holes. These are the radio, visual, x-ray, and even γ-ray emissions, which we observe on earth based systems. These emissions are radiative and depend on temperature variations within the system. In the simple case for a two temperature media, T_1 and T_2 we utilize the Stefan-Boltzmann equation for the rate of energy charge or loss,

96. $$Q = e_1\sigma\left(T_1^4 - T_2^4\right)$$

where e_1 is the emissivity at T_1 and (going to T_2) and $\sigma = 5.67\times10^{-5}\,erg/cm^2 - \deg^4$. This formalism applies to the cooler regions of the plasma surrounding the black hole.

For high energy emissions $E = \hbar\nu$ where $\hbar \equiv h/2\pi$ and h is the Planck's constant, $h = 6.683\times10^{-29}\,erg - \sec$ which applies in the x-ray and γ-ray regions. The properties of these emission frequencies, although complex, obey our previously derived scaling law [45] and the recent work of Uttley using the NASA Rossi x-ray timing explorer satellite to monitor galactic black holes for the last six years [55]. The pulsed frequency of x-ray emissions from $10^2 M_\odot$ to $10^9 M_\odot$ appear to scale in a similar manner to our by frequency vs. size [45].

The dynamic plasma media surrounding black holes become observable in the γ, x-ray, visible, and radio frequency regions by virtue of the activity within the surrounding media. Excitation modes form under thermodynamic radiative, convective and conductive processes in a charged media surrounding a rotating black hole. In order to describe this media we utilize the Poisson equation, continuity equation, and Maxwell's modified equations expressed as the Vlasov equation. Not all aspects of these media are charged as some are neutral dust, although enough charge exists to create electric and magnetic varying fields [56].

As is well known, plasma is a very hot state of matter, in which the electrons have been completely removed from their atoms, leaving positively charged ions. The ions and electrons operate freely in space. Ionized gas plasmas form plasma ionic-electron oscillations, moving in a spiral form around existing magnetic lines of force. Sometimes these plasma oscillations are called plasmons or "quanta" of collective behavior. Specific frequencies arise in particular plasma media which radiate outward and are observed by ground based observation stations. There are a number of specific oscillatory modes of a dynamic plasma medium. It acts as a fully charged fluid media, hence having properties involving Maxwell's electromagnetic description and the fluid dynamics of Boltzmann. Since it is an extremely hot charged fluid it therefore undergoes a variety of thermodynamic processes.

We proceed from our kinetic equations, which are equations of motion that describe the dynamical processes which are under examination. The picture we address divides the problem into three parts, (1) collective particle behavior in a nonequilibrium state, (2) individual particle behavior which may be in equilibrium or nonequilibrium states, and the ideal case (3) where we treat the system as an "ideal gas" of noninteracting or non-colliding particles. Then the equation of state is

97. $$E = \frac{3}{2}nKT$$

where E is the energy per unit volume, n is the number of particles, K is the Boltzmann constant, and T is the temperature in degrees Kelvin, $\beta \equiv 1/KT$. The formalism of the long range coulomb interactions characteristic of a plasma can encounter formal difficulties in the form of divergent integrals, which lead to infinities and hence singularities [5,48,57]. We denote the individual species density as n_i, n_j, etc. and the distribution function as f_i, f_j, etc., which can involve more than single particles as a distribution of "quantum like" collective states [5]. The plasma acts, in fact, more or less like a boson, in an electron Fermi field. Sometimes these collective states are termed plasmon states and can be associated with phonon or spin waves [50].

As the collective modes of the plasma are perturbed or shock excited, the distortion causes charge separation to occur in which the electric field causes the perturbations to become more apparently stable. This rate of growth of coherent states is the rate which is similar to the Rayleigh-Tayler instability caused by gravity in a semi-uniform field which can occur around collapsing gravitational systems. Under normal conditions, the gradient of the magnetic field causes the drift modes to cancel each other from the electric field perturbations but when the gravitational field is strong, drift collective states in the gravitational field may not necessarily cancel out. Hence, plasma modes can move toward or away from a black hole in their vicinity. Each charged particle in the plasma tends to carry a cloud of apparently charged particles attached by the coulomb forces and some charged particles are repelled from the cloud. The quantitative expression of the plasma system is provided by the Maxwell-Boltzmann distribution and also Poisson's equation.

For the coulomb field of particle of species i is $\phi(r)$, and the density of particles of species j with charge $z_j e$, and average density n_j, is

98. $$n_j(\underline{r}) = n_j \exp\left[-\beta z_j e \phi(\underline{r})\right] \text{ for } \beta \equiv 1/KT .$$

Then the Poisson equation is given as

99. $$\nabla^2 \phi = -4\pi e \sum_j z_j n_j = 4\pi \rho_j$$

where j represents the density of the $j^{\underline{th}}$ particle. Also basic to the plasma physics formalism is the continuity equation which expresses the conservation of charge $\frac{\partial \rho}{\partial t} + \nabla \bullet \underline{j} = 0$ where $\underline{j}$ is the vector current.

We utilize the Maxwell-Boltzmann or in some cases the Maxwell-Minkowski equations

100. $$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \nabla \times \underline{B} = \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + \underline{j}$$
$$\nabla \bullet \underline{B} = 0 \qquad \nabla \bullet \underline{E} = 4\pi c^2 \rho$$

which gives us the usual Maxwell equations as before, which are used in our plasma Vlasov formalism in which the Vlasov and Poisson equations are used [5,51]. The key is to include the electric and magnetic fields as in the Vlasov-Maxwell system [13,52].

These turbulent perturbations often diffuse and propagate transverse to the magnetic lines of force. Accounting for many higher order terms and a number of coupling constants is not directly amenable to the analytic approach and requires computer simulations. Under such variable gravitational and electromagnetic conditions patterns can emerge under cyclical interactions but also large unpredictable dynamical instabilities will occur. Our wave equations must accommodate these two cases. Some of the more detailed analytic approaches can be found in reference [5].

In general, it is considered that the net charge on stellar and galactic structures is relatively small but extreme internal charge separation can occur. The major phenomenon, however is the rotation of the system, hence the Kerr solution is utilized. The angular momentum of the system generates Coriolis-like forces and these drive convective currents. Similar type forces can drive stellar matter plasmons near black holes from ergospheric tidal action. Patterns of material and current flow can occur over "Northern" and "Southern" hemispheres which are pinched at the equator resulting in a double torus. [3]

A good example of the Coriolis effects driving collective current and soliton type structures in ionized materials is the magnetohydrodynamic behavior of weather patterns in our ionosphere and magnetosphere. These dynamics produce highly charged, self-organizing, collective and coherent activities, from hurricanes with their large field of influence, to the high energy dynamics of tornados or even ball lightning (commonly described as highly charged soliton wave patterns in a highly ionized gas, or even as self-cohering mini-black holes) [49]. Furthermore, these self-organizing structures are confined to their respective hemispheres following very specific currents that take them from the poles to the equator and back to the poles due to the Coriolis effect driven by a fundamental rotational force we defined as a spacetime torque in our earlier work [3]. This torque term may in fact be responsible for both the angular and magnetic moment of our planet, which is crucial to the dynamo effect and the production of our magnetic field.

In the Kerr and Kerr-Newman solutions, we can address the concept of radiated and absorbed energy in a collapsing system. In general, such a system is much more observable, as an x-ray and visible source, because a finite rotating event horizon exists along with a "tidally acting" ergosphere. Radiative processes can be expressed by the Stephan-Boltzmann equation, where the energy is related to temperature as $E = aT^4$ and

101. $$a = \frac{3}{5}\pi^4 \frac{R}{\nu^3}$$

where R is the gas constant and ν is the frequency. The Boltzmann constant is $K = R/A$ where A is Avogadro's number. For radiative emission, we proceed from Maxwell's equations to describe the electromagnetic forces involved in stellar black holes and their environs. One of the primary results or solutions to Maxwell's equations is to derive and describe radiation pressure. We know that light or heat propagating through a vacuum striking a reflecting surface produces pressure:

102. $$\nabla \times \underline{E} + \frac{\partial B}{\partial t} = 0 \text{ and } \nabla \times \underline{B} + \frac{\partial E}{\partial t} + j \text{ and } \nabla \bullet \underline{B} = 0 \text{ and } \nabla \bullet \underline{E} = \rho_e .$$

The mechanical force of a magnetic field creates conduction currents as radiation pressure. The current density is

103. $$j = \frac{c}{4\pi} \nabla \times \underline{B} .$$

We can establish radiation pressure and the energy of radiation, jdt, falling on the surface element $d\sigma$ of a conducting material (such as a plasma) in a time, dt. Poynting's law of energy flow

104. $$jdt = \frac{c}{4\pi}\left(E_y B_z - E_z B_y\right) d\sigma$$

the total pressure, P, i.e. the mechanical force,

105. $$F = \frac{2\cos\theta}{c} j$$

in which arbitrary radiation proceeding from the vacuum is totally reflected upon incidence on the "conductor" which we associate with the surface of the black hole's event horizon because highly charged plasma media is conductive.

The force itself is exerted on plasma particles yielding an equal and opposite momentum which is expressed in terms of the current radiation as

106. $$F = \frac{4\cos\theta}{c} j .$$

Radiative pressure is a fundamental consequence of electromagnetic energy as expressed by Maxwell's equations. Moreover this electromagnetic radiation relates directly to the Stefan-Boltzmann radiation law of the form

107. $$P = \frac{\alpha}{3} T^4$$

with a total energy

108. $$E_{TOT} = \alpha T^4 V$$

for volume, V. Through this approach and by the use of the pressure exerted by the surrounding plasma, we derive the balance equation of a dynamical black hole and its environs [56-58]. Long-range electromagnetic coherent states can be maintained under these and other highly specific conditions.

VII. HAWKING RADIATION AND THE BALANCE EQUATION

There has been a major change in the scientific thinking about the existence and nature of black holes. The "indirect" but increasingly persuasive evidence points to the existence and uniqueness of black holes. S. Hawking, R. Penrose, K.S. Thorne, and others have formulated some of these properties in terms of quantum gravity theory [35,59,60]. The usual concept is that black holes increasingly collapse to a singularity of infinite density. It was Hawking who suggested a reversal of this process might model the early universe cosmological big bang, i.e. starting from a singularity and expanding out. In the quantum gravity picture, black holes emit particles near their outer boundary, losing some energy and being reduced in size. This faint thermal radiation is termed "Hawking Radiation" [35,36]. However, in our model (described herein and elsewhere) black holes may cohere a certain percentage of the available vacuum energy, through plasma-vacuum interactions, to maintain a dynamical balance over time.

Black hole radiation is emitted somewhat in analogy to "cavity radiation" or the black body radiation problem. A container with a small hole in it, into which radiation is admitted and trapped, is reradiated through the hole when the container is heated. Likewise, radiation trapped in hot black holes also "bounces around" and is re-emitted. The properties of this radiation depend only on the temperature of the emitting system. The temperature of the cavity determines the peak frequency at which the radiation occurs. The hotter the temperature the higher the frequency of emitted radiation. The solar radiation follows the black body curve, peaking at $T = 6 \times 10^{3\,o} K$. This curve results from the Planck quantum radiation law. It is interesting to note that the quantum mechanical black body radiation model may be an analogue to the cosmological scale black hole emissions, and thus, that black hole dynamics may be fundamentally involved, and to result in, a comprehensive description of quantum gravity.

Before the advent of Planck's quantum theory, the black body radiation curve was fitted by the Rayleigh-Jeans classical law. However this law led to the so termed ultraviolet catastrophe in which the temperature vs. frequency relationship went to infinity at high frequencies. The higher the frequency, the more electromagnetic radiation would be emitted. However, Planck's quantum introduced a new universal constant, h , which yielded a fit to the observed black body radiation curve. Planck's quantized radiation emission law removed the ultraviolet catastrophe. Only a few atoms emit at high frequency values and there is a spectrum or quantum distribution of atomic emitters. Considering each radiant energy emitter oscillator to have an energy of $E \propto KT$ where K is the Boltzmann constant, for thermal equilibrium. Note that this approach has implications for the balance equation.

Then for the energy dependence on frequency by the Rayleigh-Jeans law

109. $$Ed\nu = \frac{8\pi\nu^2}{c^3} KTd\nu$$

for black body radiation. However, in the Planck hypothesis, we utilize $E = \hbar\nu$ and for the frequency range ν to $\nu + d\nu$ we have

110. $$Ed\nu = \frac{8\pi\hbar\nu^2}{c^3} \frac{1}{e^{\frac{\hbar\nu}{KT}} - 1} d\nu \; .$$

This equation yields a "Gaussian" like distribution giving a specific peak frequency of a distribution for each temperature. The peak shifts up as the temperature of the cavity goes up. The specific nature of the radiation emitters was considered in detail at the early part of the last century. Some considered the emitters to be some attribute of the aether or a set of atomic electric oscillators. With the Bohr atomic theory, the concept of atomic oscillators again took hold. However, if we are to reconsider this model in the context of black hole radiation, the simple atomic oscillator concept may require reexamination.

The black body radiation curve is an extremely important issue in the consideration of black hole radiation and in quantum gravity theories. Since the energy states in the cosmological black hole are so much greater than an ordinary "black body cavity" we may need to reexamine the emitters functions (earlier proposed to be the result of an aether), as fundamental properties of thermodynamic oscillations of a vacuum structure. To address our balance equations for black holes, we must calculate the energy / entropy associated with black holes. Hawking calculates the entropy as follows. The entropy is a measure of the number of internal states or configurations on the inside of a black hole so that it does not appear any differently to an external observer. That is, the external observer would observe no change in mass, rotation, or charge.

The entropy is expressed as

111. $$S = \frac{AKc^3}{4\hbar G}$$

where A is the area of the event horizon of the black hole, $\hbar$ is Planck's constant, K is the Boltzmann constant, c is the velocity of light, yielding the entropy, S . In terms of information theory, there is one bit of information about the black holes internal state for each fundamental area of the event horizon.

Note that the $3^o K$, more precisely, $2.7^o K$, microwave background radiation, believed to originate at the time of the big bang, obeys the black body radiation curve. What is fundamentally important is that the universe behaves as a Schwarzschild singularity. The black hole system acts as a radiative "black body" having the $3^o K$ temperature emission energy, thus the $3^o K$ radiation supports the Schwarzschild universe model [25]. We can demonstrate that the entropy of the early universe and current black hole physics is consistent with the observed $3^o K$ black body radiation. This is a significant issue in quantum gravity and the early universe and has implications for the vacuum state structure. The clues about the properties of the vacuum state occur in the early universe. Early universe models currently entail quantum gravity models. Hydrogen and helium at high temperatures in the early states of evolution of the universe involved the quantum process of pair production of electron-positron pair creation and annihilation from high energy γ -rays. This creation and destruction process is formulated in terms of creation and destruction operators, a and a^+ by Feynman graphical techniques. We relate the geometric vacuum structure into the group theory and relate to the Feynman graphical techniques in reference [5].

Black body radiation denotes complete absorption of light within a black hole. For the Schwarzschild condition the very early universe has a temperature of $T_s = 10^{32\,o}K$, T_s is temperature start of the early universe, and energy of $E_s = 10^{16}$ ergs, and E_s is energy start. As the universe evolves, T and E are inversely proportional as they evolve to present time. In the current universe condition the black body radiation is about 3^oK and the corresponding whole universe energy is about $E_n/T_n = 10^{45}$ ergs/degree. For this entropy we have $E_n = 10^{75}$ ergs where E_n/T_n is the ratio of the new energy and new time, proportional to S_n or entropy of about 10^{45} erg/degree, our inverse proportional ratio yields, for our current approximation $\frac{T_s}{T_n} = \frac{E_n}{E_s}$ which is an approximately linear relation. In fact this is only an approximation since entropy is also increasing. Note that E_n , T_n , and S_n correspond to current universe qualities [50].

A more exact approach is to consider the relationship of the temperature of the universe and the entropy of the universe throughout its evolution. This yields the inversely proportional relation of $\frac{T_s}{T_n} = \frac{10^{32}}{3} \propto \frac{S_n}{S_s} = \frac{10^{16}}{10^{-16}}$ which is ~10^{32} so $T_n \sim 1$ to 3^oK . The Boltzmann constant is given as $K = 1.38 \times 10^{-16}\, erg/^oK$ and $E = KT$. This calculation yields the current black body radiation temperature of about $T_n \sim 1$ to 3^oK which is very close to the observed value. Interstellar deep space is cold and exists near absolute zero. The interstellar media contains about one hydrogen atom per cubic centimeter of space of visible matter. We can consider other electron-electron coupling states that relate to electron-positron pair production utilizing the Bardeen-Schrieffer-Cooper method [21].

Hawking [35] and Hawking et al. [36,37] utilize the quantum theory Feynman path-integral method to describe not only the usual classical absorption but also the emission of scalar particles moving in and out of the geometry of a Schwarzschild black hole. The amplitude for the black hole to emit a scalar particle is expressed as a sum over paths connecting future singularities and infinity. Analytic continuation in a complexified Schwarzschild space is utilized to calculate the amplitude or periodicity for a particle to be absorbed or emitted [37,56,57]. The relationship between absorption and emission probabilities demonstrates that a Schwarzschild black hole will emit scalar particles having a thermal spectrum characterized by the temperature, T, as a function of mass, M , by $T = \hbar c^3 / 8\pi^2 GMk$. This gives us a simple derivative of black hole radiance [37]. The Feynman path integral is detailed in reference [5]. The usual constants are $\hbar$, c , and G and k is the surface gravity geometry of the black hole.

In the case of a Schwarzschild black hole of mass, M , then $k = \frac{1}{4}M$ using units of $\hbar = c = G = 1$. This condition between emission and absorption gives us a thermal spectrum for temperature, T , as $T = k/2\pi K$ where K is its Boltzmann constant. In a sense, a black hole is an ultimate black body radiator and this radiation is ultimately quantized. Therefore, the ratio of emission is equal to that of absorption for N photons in a cavity with energy E , is $e^{-E/KT}$, then the condition for equilibrium that is balance is $T = k/2\pi K$. Hence, Hawking radiation is key to our formalism of the balance equation. We also observe that we return to the relationship between the Rayleigh-Jeans Law of Planck's quantum formalism.

Hawking concludes that this thermal emission leads to a slow decrease in the mass of the radiating black hole [35]. Primordial black holes of less than 10^{15} gm could evaporate to a Planck mass size of 10^{-5} gm singularity in the current universe [49,58]. In this paper we detail closed cosmologies and the Schwarzschild solution. However, in Hawking's work, certain critical assumptions are made regarding quantum gravity theories and the conditions of Einstein's field equations. For example, in our work we take into account a torque term in the stress-energy tensor [44]. Currently, there are no consistent theories of quantum gravity. One such assumption made by Hawking [35] is the use of a flat Minkowski metric consistent with classical mechanisms.

Hawking uses the standard second law of thermodynamics requiring that entropy never decreases. The entropy of matter outside the black hole is the sum of the surface area of the event horizon, k . As gravitational collapse proceeds, the entropy of baryons and leptons in the collapsing body creates entropy which is supplied to the plasma outside the event horizon.

We can consider that the probability amplitude for a black hole to emit particles is related to the probability of the black hole to absorb particles. Hawking [36] also gives us a form of quantum gravity by his use of the path integral method as utilized by Rauscher [5] in the description of quantum plasmas. Hence, our balance equation is formulated in terms of quantum gravity in the expression of strong and electroweak forces. This is because the strong gravitational field of the black hole yields a superdense plasma [61].

The path of a particle is formulated in terms of path integrals in curved space [5,36]. From the use of the path integral method one can derive an inhomogenous wave equation in a Schwarzschild field. As done in reference [5], a Feynman propagator $k(x,x')$ is formulated, where a particle moves from a spacetime point x at $t=0$ to x' where $t>0$ or t_p for an action integral

112. $$S[x(t_p)] = \frac{1}{4}\int_0^{t_p} dt g\left(\dot{x}\dot{x}'\right)$$

where g is the metric on a curved spacetime and $\dot{x}$ and $\dot{x}'$ represent tangent vectors with components

113. $$\frac{dx^\alpha}{dt} \text{ and } \frac{dx'^\alpha}{dt} .$$

The path with extremes S satisfies the geodesic equation with t as an affine parameter. The timelike paths t are a constant multiple of proper time and the space-like paths are taken as the same multiples of proper distance. This gives us the relativistic part of the formalism. A black hole can be considered an electrically conducting spheroidal membrane at the event horizon. Its energy of rotation or angular momentum is expressed as large electric current flow, extreme gravitational interactions, and the angular momentum itself. A black hole of a given mass has a maximum rate of rotation in which there is a balance between the imploding and accreting matter so that the centrifugal forces are in balance. The centrifugal force, due to torquing, counteracts the inward pull of gravity and prevents more matter from falling into the black hole, which would further amplify its spin. For example, a black hole of 10^8 $M_\odot$ or 100 million solar masses, yields a rotational energy of about 3 x 10^{48} kilowatt-hours. This energy feeds the Hawking radiation field, and the charged-rotating Kerr-Newman system produces a charge field that feeds energy into the surrounding plasma. It has been hypothesized that such a black hole system explains quasar phenomena by creating the energy emitted by quasars. The energy that quasars emit is on the order of an entire galaxy. It is possible that the black holes at galactic centers accrete enough matter to overwhelm the whole galaxy.

The torquing of spacetime affects matter-energy in a vicinity of space such as to create the galactic rotational form we observe as external elements of the galactic central black hole. A rotating charge creates a current flow around and in the charged plasma media produce magnetic fields. The magnetic field lines excite surface eddy currents and become distorted by the black hole rotation. They can pinch off as loops in the plasma field somewhat in analogy to solar coronal ejections and sunspot eddies of the solar plasma field. A rotating, charged black hole can act like a giant battery having an enormous voltage drop between its poles and its equatorial region of as much as 10^{20} volts. Thus, the black hole acts as an enormous dynamo. As the magnetic field lines cross the polar field lines and return to the rest of the black hole, the induced current flow deposits its energy in the intervening plasma and accelerates it outward. This process, driven by spacetime torque, explains the jets of gas observed emanating from quasars, galaxies, and supernova poles and their enormous luminosity. The dynamical jet formations merge into the galactic halo.

The interstellar gas and gas near the event horizon become heated and, like any hot gas, emit radiation in radio, visible light, and x-ray bands. The Hawking radiation also involves the emission of trapped particles. Hence, black holes store tremendous energy through their gravitational field, charge field interaction, and spin. R. Penrose noted that the storage of energy in the rotation of a black hole is significant and may lead to the quasar and supernova stages. The significant determinant of a relatively stable galactic centered black hole, quasar, or supernova depends on the intrinsic balance between the torquing and the structure of the black hole, and the properties of surrounding plasma described in the balance equation.

VIII. EXPERIMENTAL EVIDENCE AND OBSERVATIONS OF BLACK HOLES AND SUPERFLUID STATES OF MATTER.

Black holes are dynamically rotating systems. We present examples of such systems here and in reference [5]. Also there is emerging experimental accelerator evidence of mini black holes, and we present a brief discussion of some of this work here and in reference [3]. In addition we discuss some laboratory plasma experiments relating to the dynamic media hypothesized and observed to surround black holes.

Cygnus X-1 in the constellation Cygnus, the Swan, is an intense x-ray source and is observed as a distant blue star. It moves around a dark member identified as the first stellar black hole. The x-rays are emitted as matter from the blue star is accreted by the black hole. In the spring of 2001, the Rossi x-ray timing Explorer satellite made recordings of skewed variation in light emitted from GROJ1655-40 and, like distant quasars, atomic fragments are emitted at right angles to a disk of hot spinning gas. This system lies within the Milky Way. The 40 in GRO's name refers to the distance of 40 miles above a black hole of $7M_{\odot}$, which produces an observed brightness variation of 300 times per sec. The 40 miles correspond to the distance from the black hole event horizon to its orbiting plasma material. Also a 450 Hz signal is observed in the variation of emitted light, indicating a layer of material approximately 30 miles above the event horizon. This close proximity of orbiting material indicates the black hole and external plasma are rotating and charged, indicating a Kerr-Newman metrical description. These and other observations give strong credence to our approach.

Additional observations were performed with the Hubble Space telescope, the Extreme Ultraviolet Explorer (EVE) Satellite and the Chandra x-ray observatory of XTE j1118x480 which orbits a star of about one solar mass, $M_{\odot}$. The spiraling disk of plasma orbits at about 600 miles from the ergosphere. This more distant orbiting material appears to be superheated, which produces the larger orbiting radius.

These systems act like an energetic charged field torquing engine driven by the high gravitational field of the black hole. Einstein's field equations do describe black hole physics but do not include the effects of strong and weak forces or the electroweak force and, in general, electromagnetic forces for astrophysical systems. It is these forces that act to stabilize cosmological and astrophysical systems against complete gravitational collapse, which allows our balance equations to hold.

Relativistically, as the gravitational forces from black holes increase during collapse, time dilation occurs. Hence, we externally observe the process of collapse in the observer's frame of reference, as exceedingly slow. From that point of view, the black hole acts as a giant dynamo of gravitational electromagnetic and nuclear processes producing a relatively stable and dynamically balanced system. Because both gravitational and electromagnetic forces are long range, a dynamic balance between these two forces is of primary significance, and it includes both the structure of black holes and their surrounding media. Gravitational torque and Coriolis driven magnetic forces shape the media surrounding the black hole. These forces, in balance, create the paths upon which the charged particles move, creating a toroidal form of the plasma media. As the "polar" jets emerge, part of the plasma loops back towards the equator to create a dual toroidal topological form. Spin and charge forces are necessarily considered to explain this formation and, as we have demonstrated, the Kerr-Newman solution including torque and Coriolis effects predicts such a formation [3].

The pulsation of the electromagnetic field from 200 to 600 Hz in the previously observed quasar systems produces a pulsation in the lines of force of the magnetic field. Therefore, a set of resonant frequencies occurs in the system. The form and frequencies emitted by the dynamic plasma are also determined by the effects of nuclear strong and weak forces. These short range forces also have intermediate range effects through their coupling to the electromagnetic force as the electroweak force and through supersymmetry. Collective coherent states arise out of the plasma from short range interactions of atomic level couplings as we have shown earlier here and in reference [5]. Under extreme gravitational conditions, strong and gravitational forces may become balanced as presented in reference [3,43,62].

The emission of Hawking radiation is to be examined at the CERN Hadron Supercollider that is expected to come online in 2007. The multipurpose accelerator will be used to look for the Higgs boson as well as possible indication of mini black hole production. Detection of the possible production of mini black holes is expected to be observed through Hawking radiation and by other means.

Experiments conducted at the Brookhaven Relativistic Heavy Ion Collider (RHIC) are thought to have produced an analogous entity to a black hole. Nastase, et al. [63] compares the RHIC produced fireball to a black hole, but the fireball is too small and too low in energy to accrete matter from surrounding media. By use of Einstein's equivalence principle, Tuchin [64] of the Brookhaven Theoretical Nuclear Physics Group, considers that the acceleration and deceleration of ion beam collisions with target atoms can be considered to act as the extreme gravitational conditions similar to that of a black hole. Reaction times are of the order of the strong interaction time of 10^{-23} sec. Such a picture appears to be reasonable if Hawking-like radiation was detectable. Such radiation would prevent long lived mini black holes from existing because energy for their existence is carried away in this radiation.

Heavy ion two-beam collision experiments at the Brookhaven RHIC using gold ions, stripped of most of their electrons have a total energy per collision of 40 TeV in the center of mass reference frame. Also, surplus shock energy of the order of 25 TeV produces a fireball which creates new particles [65]. As many as 10^4 new particles are produced in these fireballs. Analogy is made to the early universe conditions in which the first atoms are formed for $t_U \sim 4x10^5$ years. The size of the fireball that is produced is about $5x10^{-10}$ cm or about $.05x10^{-8}\ \text{cm} = .05\ \dot{\text{A}}$.

Another informative result of the recent experiments conducted at the RHIC is that the fireball produced by the high energy collisions acts as a quark-gluon liquid with fluid properties, such as can be treated as having capacity and viscosity. These results do not agree well with the standard model which expects gas like behaviors of particles at these scales. However, shockwaves are produced as one would expect in liquid systems. The behavior of particles in the RHIC collision experiments appear to act as a very low viscosity fluid and hence as a superfluid [65,66]. These systems may obey the standard laws of hydrodynamics and, under the influence of external magnetic fields, act as an MHD medium. Since the early universe of the pre-protonic era may have undergone an evolution of cold Fermion and Boson states that acts as fluids, we can term this new field of study a *fluid dynamical MHD cosmology*.

In his work at the RHIC on the produced fireballs, H. Nastase, et al. [63] argue that the observed fireballs form an analogy to a pair of black holes. This pair would comprise a double torus configuration [63]. Inside the black hole fireballs, Nastase hypothesizes that deconfinement of quarks and gluons occur. Such a state of matter has been hypothesized for the early universe. It appears that we have a relationship between the black hole state and superfluidity. In Section IX, we present a possible relationship between plasma dynamics and quantum electron interactions, and electron coupling in the BCS model.

Recently, atomic experiments have been conducted on an ultracold gas of Lithium 6, Li^6, atoms which can act as a vibrating semi liquid or as a superfluid. Laser beams are used to heat cooled atoms in which vortices are observed, indicating a superfluid state. The gaseous Li^6 cooled state represents Fermion supercooled, superfluid media. One other superfluid has been found for liquid Fermion atoms of Helium, He^3, which is also superconducting. Such superfluid superconductors may have properties similar to neutron stars.

In the Li^6, experiments a superfluid gas, rather than a liquid, more closely matches the material density of the interstellar medium. Under the conditions of an applied external field the Fermionic superfluid state acts as Fermi atoms forming pairs before producing the Bose-Einstein condensate [67]. The usual Cooper pairs found in superconductors involve the BCS Model [68] and are not the same as the new states observed in the Li^6 atoms. Also, under resonant pulsed application of the laser trapping of the Li^6, where an external electric field was applied to confine the atoms, a frequency of vibration of the media was observed to be 2837Hz, which is close to the theoretically predicted frequency of 2830 Hz for the hydrodynamics of a Fermion gas. An analogy to Copper pairing [22] is made for spin up and spin down atoms. All the details for the theoretical model of the Fermion state are not worked out [68]. Spin coupling in superfluid states allows us to treat an ultracold Fermion gas, such as Li^6 in analogy to a Bose-Einstein condensate for electrons. The state of superfluid matter is always found to be vorticular in structure and persists in frictionless flow [69]. Since this superfluid phase is of the order of density of the interstellar atomic matter, clues may be found for observed astrophysical forms [3].

Perhaps we can form a model which relates the superfluid vorticular flow to observed vortices emanating from quasars to supernovae. Exposure of the superfluid gas to externally imposed magnetic fields can form a tuned resonant system state with the Bose-Einstein condensate and/or related supercoherent states. We have a possible relationship of superfluid matter and black hole physics [66]. In addition, we have a balance system comprised of the relationship of black holes and their surrounding plasma media. In the next section we will examine theoretically the relationship of the plasma MHD equations and the supercooled, superconducting BCS model. This relationship further describes the long range coherent states of matter which surround black holes. Rotational or vorticular flow in a superfluid has zero curl, unlike a solid, because the flow velocity is the gradient of a phase. The superfluid as a whole will respond to frame dragging created by vorticular flow and between vortices, the flow is irrotational or $\nabla \times \underline{v} = 0$ in the superfluid condensate.

IX. ELECTRON COLLECTIVE COHERENT STATES IN PLASMAS AND IN SUPERFLUID STATES IN THE MEDIA SURROUNDING BLACK HOLES: THE RELATIONSHIP OF THE MHD AND BCS MODELS

The results of the fireballs, of visible but small size ~0.1 mm, from the Brookhaven National Laboratory Relativistic Heavy Ion Collider have yielded unexpected results. The heavy ion (gold) collisions on gold were designed to produce a media that mimicked early universe conditions as described in the previous section. Instead of producing a quark-gluon plasma which behaved like a charged gas, with independent particle motions, collective coherent states within the fireballs were observed [66]. The particles collisions moved collectively under the influence of pressure variations in the same manner as a fluid state. The post collision particles behaved like a perfect uniform fluid, in which heat was able to dissipate rapidly.

In this section we will demonstrate that the collective behavior that arises in plasma and certain liquid and superfluid state phenomena, may not only explain the results of these and other experimental results but also yield information about early universe conditions (~$14x10^9$ years ago). These fundamental collective states also explain the properties of the media surrounding black holes universally, from mini black holes, stellar black holes, and galactic black holes. Ultimately, the formulation of the dynamics of these states may lead to a more complete understanding of the properties of spacetime itself acting at all size scales, including the structure of the vacuum. We will observe that our formalism explains the ubiquitous nature of these collective coherent states.

The relationship between the plasmon-acouston (acouston-like phonon) states of a plasma are related to the exciton modes and Bose-Einstein Condensate (BEC) states in a gas of indistinguishable atoms. One commonality of these states is the collective behavior of individual particles. In these modes of excitation-interaction, the individual particles, electrons, ion or atomic wave functions, extend in space and overlap when the individual particle states cohere in a nonlinear media and are close enough, i.e., space below the Debye limit.

The BEC states, obeying Einstein-Bose statistics, can be formed at low temperatures of several ^oK systems, which reduce the kinetic energy states of the atoms to the order of interstellar space media. Then the atomic matter waves overlap and become coordinated so that collective macro particles and atomic states arise in the media. The formation and effect of collective, coherent states of the media is such as to polarize the vacuum states. It is only through the collective states that the effect of the vacuum can be detected and measured and produce macroscopically observed effects. Some of these effects entail the structure and properties of the vacuum which determines the formation of these states in matter.

One method of producing BEC states in the laboratory is by energy supplied by infrared laser beams, which bombard target atoms. The laser frequencies are chosen so that their photons cool down atoms by bombardment and hence act to cool down the atoms to a few degrees Kelvin. This laboratory procedure also employs magnetic cooling techniques which can be used to selectively “incorporate” more energetic atoms to further cool the system. These BEC states are long-lived, can last up to twenty seconds using intermediate mass atoms. Likewise, the conditions around stellar and galactic black holes comprise states of matter that, under strong gravitational and magnetic fields, in highly luminous condition, also form very coherent, collective states of matter. In previous sections we have enumerated some of these states in our formation of plasmon, phonon, and exciton states and their effect on polarization of the vacuum.

The exciton state is a state of the vacuum in which an energetic electron and a positron state formed from a hole in the vacuum Fermi sea through photon decay to the e^+e^- pair. In the case of exciton formation, the electron and positron can temporarily orbit each other which is called a singlet exciton. Multipart excitons can be formed and represent one of the forms of vacuum excitation. Excitons were first considered as specific states in certain crystal lattice formations. It has been determined in laboratory conditions that singlet excitons have no magnetic moment, but triplet excitons have a magnetic moment.

We will demonstrate the manner in which the magnetohydrodynamic equations and the Bose-Einstein condensate equations relate to each other. This gives us a significant formalism to relate the particle fields surrounding apparent holes and their fluid-like properties. The collective states that arise out of these equations yield soliton solutions. These solutions, which can be related, are derived from a nonlinear Schrödinger equation [57] and can be identified with quantum correlations when the quantum coherence length becomes comparable to the length scale over which the supercoherence or superfluid density varies [70]. One such circumstance where this occurs is the center of superfluid quantum vortex in a rotating fluid hydrodynamic system. Some of these coherent states can be sustained over macroscopic dimensions.

Elsewhere we explore in detail a nonlinear Schrödinger equation which utilizes the potential energy of torque and its kinetic effects such as the Coriolis force yielding soliton solutions with spin dynamics [8]. This approach yields a quantum gravity picture with a fundamental rotation or spin component in the metric tensor [3]. In the case of rotating spacetime which arises from the fundamental torquing component in the metric, we have a line element of the form

114. $$ds^2 = g_{oo}dt^2 + g_{oi}dtdx^i + g_{ij}dx^i dx^j$$

where $g_{\mu\nu}$ is the usual form space metric with indices μ,ν running from 0 to 3, and indices i, j run 0 to 2 and g_{ij} is the Haramein-Rauscher metric [3]. We can also write the four space metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa_{\mu\nu}$ where $\eta_{\mu\nu}$ can be identified with the Coriolis effect and $\kappa_{\mu\nu}$ represents the effect of the background spacetime, which introduces a perturbative dynamical term in the Lagrangian,

115. $$L = \frac{1}{2}\mathrm{T}^{\mu\nu}\mathrm{T}_{\mu\nu}$$

where $\mathrm{T}_{\mu\nu}$ is the symmetrical stress-energy-momentum tensor for the coordinates. The usual spacetime coordinates are (X,t) or (x,y,z,t) in which the usual term is utilized and has the metric $g_{\mu\nu}$. The expanded space is expressed in the coordinates (X,χ,t,τ) which covers a larger domain. We will expand on this concept later in this section and in reference [8].

A complete formalism, which relates the plasma field surrounding black holes and their excited fluid state conditions, has not been completely formulated. Our approach in this section is to relate the MHD and BCS type approach under the field conditions of strong gravity and large electromagnetic fields. The key to this formalism is the effect produced in the vacuum and the vacuum's effect on the media [69,71-73]. In this section, we examine the mechanism of superfluidity and superconductivity, the BCS model. Cooper electron pair coherent states, and other exciton models [72,73]. We will examine the relationship of these exciton models to the phonon or plasma state described in the previous sections and to the solitary wave collective state. The relationship between the quantum coherent excitation states in the MHD physics and the Cooper pair electron states have interested Rauscher [72] for a number of years. Currently laboratory and astrophysics data have borne out this connection. Collective electron behavior is key to coherent states in plasma and superfluid systems as well as lattice coherent states in solids. It is possible that such an approach may allow us to develop "room temperature" soliton conductors, i.e., apparent thermal superconducting-like states. These states may naturally exist in the dense plasma media surrounding black holes. The relationship of the MHD approach and the BCS approach to excitons can give us insight into plasma coherent states.

The detailed mechanism of superconductivity and plasma oscillations are similar. The plasmon or phonon oscillation arises because of the dynamic behavior of the electron gas in both our quantum plasma theory and the BCS theory. Collective electron behavior of the electron gas is activated when an external electric and magnetic field is applied. Collective states activate, excite, and polarize the vacuum. Since the electrons, possessing a finite mass and thus inertia moves in such a manner as to screen static electric fields, the electrons "over-shoot" the field. This process produces a net positive field by the electron's absence in some fixed location instead of a negative net charge. The collective electron behavior produces another nonequilibrium distribution and, in a sense, produces an opposite electric charge. The electrons then begin to move in the reverse direction and again over-shoot their equilibrium target, and so on. The electrons produce an oscillatory motion with the Coulomb interaction acting as a restoring force with the mass of the electron as the inertia. This process is called a plasma oscillation and is related to the Debye screening process and determines the size of the coherent oscillations in a particular set of external field conditions. The key is the production of nonuniform, nonequilibrium space-charge filed in the electron gas and the properties of the externally applied field of strong gravity and electromagnetic fields.

The plasma interaction is not accounted for by the simple picture of binary collisions. The potential field around a charged particle is effectively screened by the cloud of other charged particles. The characteristic domain of this cloud is determined by the plasma conditions such as density and temperature. This distance is termed the Debye screening length, λ_D as we previously discussed.

For the case of a metal lattice, for example, the electron-electron interaction and field effect is produced by the screening process. The effective interaction is short-range and of the form $(e^2/r)e^{-kr}$. Here, we consider the electron-electron as well as the electron-ion interaction. The electron-electron interaction is directly repulsive in the bare Coulomb interaction.

In the metal lattice or superdense media in space, interaction is mediated by an "over-shooting" process. As a current flows and the electrons are in motion, positively charged ions begin to move toward the instantaneous position of the electrons which are moving at the Fermi velocity, v_F, where $v_F > v_i$, v_i being the ion velocity. Thus, there will be a "packed" region behind the electrons containing an excess positive charge density. This positive density region attracts a second electron and hence, there is an apparent positive electron-electron interaction. This attraction is indirect and is termed the phonon-mediated electron-electron interaction and is the screening interaction between electrons and ions and between electron-electron interaction states which acts as a collective coherent motion in the media. In the simple two-component plasma consisting of equal densities of electrons and ions, the oscillation of the plasma frequency corresponds to an out-of-phase oscillation between the two constituents, which is termed the optical mode of the plasma, which is sometimes associated with optical pumping of the plasma. The in-phase oscillations are termed the acoustic mode of the plasma and these are the ones of interest to us.

The basic BCS theory of superconductivity is one of the most striking examples of a successful theory of solids [21]. The BCS theory, or the independent quasiparticle approximation, has been applied to the theoretical formulation of superconductivity as well as to nuclear matter single-particle states. The BCS theory accounts for most of the observed experimental effects associated with superconductivity. In the isotope effect, the critical temperature, T_c (for the onset of superconducting properties of an element), is related to the isotope mass, m, as $m^{1/2}T_c$ = constant, for a given element (except for the transition elements Rubidium, Ru, and Osmium, Os). This result suggests that the properties of the lattice phonons, zero-point or thermal, are involved in superconductivity and thus explains the dependence of atomic mass. It is believed that the interaction responsible for superconductivity is the attractive interaction between two electrons near the Fermi surface, caused by their interaction with the zero-point phonons. In the case of the elements Ru and Os, which do not show an isotope effect, the interaction with zero-point electron modes is not dissimilar to anti-ferromagnetic domains or the Meissner effect in which the magnetic field is excluded [74].

The discovery of flux quantization, in units of one-half the natural unit ch/e is a strong confirmation of the central role of paired electron states as predicted by the theory, where c is the velocity of light, h is Planck's constant, and e is the electron charge. There are possible ambient "room-temperature" coherent collective phenomena that may exhibit similar properties to that in superconducting materials, which may be identifiable as soliton states [75,76]. This research yields a relationship of laboratory experiments derived from the understanding of stellar and cosmological black hole media.

The short range part of the two-body interaction force is approximated by the so-termed pairing force that couples two particles (electrons or ionic nuclei) with total angular momentum equal to zero. Higher order (long range) terms can be included as a multiple expansion in order to account for coupling two particles with nonzero angular momentum. The Hamiltonian for the problem is

116. $$H = H_0 + H_{\text{int}}$$

where

117. $$H_{\text{int}} = H_{pair} + H_{qp}$$

and

118. $$H_{pair} = -\frac{G_0}{4}\sum_{v_1}\sum_{v_2} a^+_{v_1} a^+_{-v_1} a_{-v_2} a_{v_2}$$

for interaction-Hamiltonian H_{int} and pairing force interaction H_{pair}, and $a^+_{v_1}$ and a_{-v_2} are the Fermi operators that create and destroy a particle in state v. The interaction couples the individual particle states to the collective phonon states and the coupling of the two electrons occurs indirectly through the phonon field. The phonons occur in plasma excitations as well as condensed matter excitations. We utilize the Hamiltonian formalism in this section and the next section to describe the collective states of matter and the vacuum structure.

Using the second quantized formalism, we can formulate particle-hole interactions of elementary excitations. The completely occupied Fermi sea extends to the vacuum state. In general, the superposition of particle-hole states and the vacuum state corresponds to a modification of the Fermi sea negative energy states of the vacuum, and hence, the vacuum occupies a constant role in these collective state phenomena. An analogy can be made to vacuum structure and crystal structure, and hence, we may be able to deduce the lattice structure of the vacuum. We calculate the excitation energy for various crystal deformations, ε, with higher order multiple interactions. In analogy to the band layer electrons, we can calculate the gap energy with this model. The energy-like gap parameter is denoted as Δ and single-particle energies as ε_v. Two-particle interactions are given in terms of the pairing force interactions where the pairing force is given as G_0, the coupling parameter in the Hamiltonian.

The key to the 1957 BCS superconducting model is the concept of the formation of Cooper pairs of electrons, which arise from the electrons in a conductive media, whether a solid, liquid, or gas [21]. Each member of the pair moves in the opposite direction at the same speed. If no current from an external electron or magnetic field is applied, the center of mass of the electron pair is zero. If a current is applied, then the effective center of mass is not zero, and hence a net moment arises. We might say that the ion oscillation acts like an electron-electron interaction pump. This vibration gives rise to an electron pair acceleration by the imbalance in the vibrations of the ions which produce a net charge effect and hence mediate the electron-electron interaction. This is similar to the electron collective states in a plasma. The key here is to understand the relationship between plasma dynamics, superfluidity, and superconductivity, which appears to indeed come together in current high energy physics experiments and in the vicinity of galactic nuclei.

In the highly organized superconducting superfluid state, a change in the momentum of one pair requires a change in the momentum of all other pairs as well. The energy needed to redistribute the momenta of the Cooper pairs, which creates the electrical resistance, is much larger than the vibrational energy available in the lattice at low temperatures. In the laboratory, the critical temperature at which we have the onset of superconductivity is variable. For example, for helium (the first superconducting material found by H.K. Onnes in 1911) it is $T_c = 4.2^o K$; for the metallic alloys we have $T_c = 17^o K$; for vanadium-silicon and niobium-aluminum-tin it is $T_c = 18.7^o K$; and $T_c = 23.2^o K$ for an alloy of niobium-germanium (Nb_3Ge) and $T_c \sim 93^o K$ for Y Be Cu O.

We detail the second quantized formalism for superfluid, superconducting systems which are similar to the formalism for the field-theoretic plasma calculation which has been presented already. This approach well describes the collective states of a many-body media for cool plasma and superfluid states. With our Hamiltonian approach, we describe photon coupling and electron-electron interaction and compare these states to electron-positron states arising from the vacuum. We use the second quantized formalism, as before, for the indirect electron-electron coupling through the phonon field. To the first order for the electron-photon Hamiltonian we have

119. $$H' = \sum_{\underset{\sim}{k}\underset{\sim}{q}} A C^{+}_{\underset{\sim}{k}+\underset{\sim}{q}} C_{\underset{\sim}{k}} \left(a_{\underset{\sim}{q}} + a^{+}_{-\underset{\sim}{q}} \right)$$

where C^+ and C are the Fermion (electron) creation and destruction operators and a^+ and a are the operators for boson or phonon fields, and A is a time-ordering C-number [5].

The effective Hamiltonian for the interaction between two electrons is given by the second-order perturbation theory, H'':

120. $$H'' = (-H)\frac{1-p_0}{H_0 - E_0}(H)$$

where $H \equiv H_1' + H_2'$ for the two electrons and H_0 is the unperturbed Hamiltonian and E_0 is the ground state energy, and the $\frac{1-p_0}{H_0 - E_0}$ is the perturbation operator. The subscripts k and q define electron and photon states, respectively.

Ignoring self-energy terms and taking the expectation values of H'', at absolute zero, terms in a^+a vanish and aa^+ give unity, so that

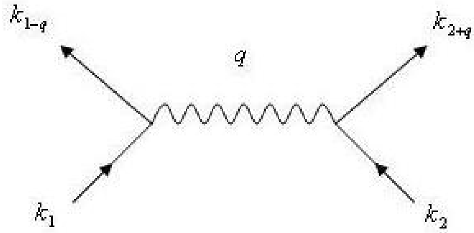
121. $$H'' = D^2 \sum_{\underset{\sim}{q}} C^{+}_{\underset{\sim}{k}_2+\underset{\sim}{q}} C_{\underset{\sim}{k}_2} C^{+}_{\underset{\sim}{k}_1-\underset{\sim}{q}} C_{\underset{\sim}{k}} \left\{ \frac{1}{\varepsilon_{\underset{\sim}{k}_2+\underset{\sim}{q}} + \omega_{\underset{\sim}{q}} - \varepsilon_{\underset{\sim}{k}_2}} + \frac{1}{\varepsilon_{\underset{\sim}{k}_1-\underset{\sim}{q}} + \omega_{\underset{\sim}{q}} - \varepsilon_{\underset{\sim}{k}_1}} \right\}$$

Considering the diagonal elements of H'', energy is conserved between the initial states k and final states q. The k states, $\underset{\sim}{k_1}$ and $\underset{\sim}{k_2}$ refer to the two electrons involved in the interaction. For our Hamiltonian, we have

122.
$$H'' = D^2 \sum_{\underset{\sim}{q}} \frac{2\omega_{\underset{\sim}{q}}}{\left(\varepsilon_{\underset{\sim}{k_1}-\underset{\sim}{q}} - \varepsilon_{\underset{\sim}{k_1}}\right)^2 - \omega_{\underset{\sim}{q}}^2} C^+_{\underset{\sim}{k_2}+\underset{\sim}{q}} C_{\underset{\sim}{k}} C^+_{\underset{\sim}{k_2}-\underset{\sim}{q}} C_{\underset{\sim}{k_1}} .$$

The electron-electron interaction for this equation is attractive, that is through negative charge states for excitation energies of $\left|\varepsilon_{\underset{\sim}{k}\pm\underset{\sim}{q}} - \varepsilon_{\underset{\sim}{k}}\right| < \omega_{\underset{\sim}{q}}$, and in other cases repulsive in nature. The Feynman graph for H'' is

123.



which represents the electron-electron indirect interaction through the lattice or coherent phonons (in analogy to a laser). Even in the attractive region, the interaction is opposed by the Coulomb repulsion, but for sufficiently large values of the interaction constant, D, in our expression for H, the phonon interaction dominates when $\left|\varepsilon_{\underset{\sim}{k}\pm\underset{\sim}{q}} - t_{\underset{\sim}{k}}\right| < \omega_D$ where ω_D is the Debye energy. That is where most phonons are near the Debye limit. In this limit we can simplify it as

124.
$$H'' = -V \sum_{\underset{\sim}{q}} C^+_{\underset{\sim}{k}+\underset{\sim}{q}} C_{\underset{\sim}{k}} , C^+_{\underset{\sim}{k}-\underset{\sim}{q}} C_{\underset{\sim}{k}}$$

summation over final states $\underset{\sim}{q}$ are made, and the quantity, V, is taken to be a positive constant or $V > 0$. This Hamiltonian contains the essential features of the problem. Additional terms in a^+a come into the Hamiltonian for $t_c > 0^o K$.

In order to understand the superconducting-like state we need to understand the properties of a Fermi gas under an attractive two-body interaction with the Debye cutoff and the role of Cooper electron pairs. As we before stated, the Fermi gas directly interacts with the vacuum state. These structures could not exist without an active structured vacuum. Bound state electron pairs are fundamental to the formation of the collective states in a Fermi gas. L. Cooper [22] was the first to suggest that unusual coherent properties would arise from the attractive interactions in a Fermi gas. He proved that the Fermi sea is unstable against the formation of bound pairs. This discovery led directly to the BCS superconducting state analysis and also relates to MHD collective electron states. The BCS theory deals with the many-electron problem in the second quantized formalism which is more complex than the electron pair problem, but the pairs are crucial in a very important manner to the BCS matrix element. Because the density of the superconducting electrons is of the order of $\rho \sim 10^3$ electrons per cm^3 or more, Cooper pairs would have to occupy the identical volume. This high degree of overlap is so great that we can consider the superconducting ground state as a collection of non-interacting pairs.

We calculate the eigenfunctions and Hamiltonian for two electron states. For a center-of-mass system, we have $\underset{\sim}{k_1} = \underset{\sim}{k}$ and $\underset{\sim}{k_2} = -\underset{\sim}{k}$, so that the one-electron state is the pairs $\pm\underset{\sim}{k}$. Including the electron-electron interaction (our last expression for H''), we have

125.
$$H = \frac{1}{m} p^2 + H''$$

for $2p^2 = p_1^2 + p_2^2$ with the eigenfunction form

126. $$\phi(x) = \sum_{k} f_k e^{ik\cdot x} = \sum_{k} q_k e^{ik\cdot x_1} e^{-ik\cdot x_2}$$

for $x \equiv x_2 - x_1$ and $k = 1/2\left(k_1 - k_2\right)$. For eigenvalue equation $(H - \lambda)\phi(x) = 0$ with similar equation for $\varepsilon_k = k^2/m$, $\left(\varepsilon_k - \lambda\right)g_k + \sum_{k'} q_{k'}\left\langle k_1 - k\left|H''\right|k_1' - k'\right\rangle = 0$ for our matrix element with $k = k' + q$ and $-k = -k' - q$. The density of two electron pairs in the states $k, -k$ per unit energy range is defined as $\rho(\varepsilon)$ and the secular equation, for the Lagrange multiplier λ is given as

127. $$(\varepsilon - \lambda)g(\varepsilon) + \int d\varepsilon' \rho(\varepsilon')g(\varepsilon')\langle\varepsilon|H''|\varepsilon'\rangle = 0$$

where we take V as positive or $V > 0$ and can express it as $-V = \langle\varepsilon|H''|\varepsilon'\rangle$. The energy range $\pm\omega_D$ of one electron relative to the other of the pair outside of this energy range is zero. In this manner, we can define the energy states for the onset of superconducting properties. In the momentum space k we can suppose a pocket made up of one-electron states above the top of the Fermi sea, between ε_F and $\varepsilon_F + \omega_D$, or between k_F and k_m where k_m is defined by

128. $$\omega_D = \frac{1}{2}m\left(k_m^2 - k_F^2\right).$$

We can approximate $\rho(\varepsilon')$ by ρ_F, the constant value of the Fermi level, and define the lowest eigenvalue as λ_D and $\lambda_D = 2\varepsilon_F - \Delta$ for

129. $$\Delta = \frac{2\omega_D}{e^{1/\rho} F^{V-1}}$$

where Δ is the binding energy for the pair relative to the Fermi energy level. This energy gap can depend on temperature. If V is positive, we lower the energy of the system by exciting a pair of electrons above the Fermi level, and since this excitation state is more stable, which implies that the Fermi sea is unstable, it has intrinsic properties that give rise to collective particle pair and other states. These "band layer electron pairs" give us the superconducting properties.

The ground state of a Fermi gas has attractive electron-electron state interaction. This state is associated with a superconducting ground state, and in the Hydrogen and Helium dominated interstellar media, can be associated with vorticular flow in superfluids as well as collective particle states in plasma. For one-electron Bloch energy, ε_k, relative to a Fermi level energy of zero, the complete Hamiltonian is given by

130. $$H = \sum_{k} \varepsilon_k C_k^+ C_k - V \sum_{q} C_{k'+q}^+ C_{k-q}^+ C_k C_{k'} \ .$$

For simplicity, we do not denote spin indices. The Pauli exclusion principle needs to be considered carefully in the sums. This is the standard form of the BCS Hamiltonian, similar to the BCS reduced Hamiltonian. See (124).

There are a large number of nearly degenerate configurations in which the Hamiltonians relate to each other. If all the terms in H'' are negative, then we obtain the lowest energy state for the Cooper pairs. Usually, without Cooper pairs, there would be as many positive as well as negative matrix elements of V and no superconducting state would occur. We can generate coherent states of low energy by use of the subset configuration between matrix elements of the phonon field interaction for $V > 0$; then the Bloch states are always occupied in pairs. The interaction conserves the wave vectors, by unitarity, such that we can examine only pairs which have the same total momentum $k + k' = K$, where $K = 0$ for a rest center-of-mass and the pair is denoted as $k, -k$. We have not as yet introduced the electron spin; for antiparallel spins, spin-up or spin-down pairs, where the energy will usually be lower. We shall denote the spin index as k for spin $\uparrow$ up $-k$ for spin $\downarrow$ down.

Our BCS reduced Hamiltonian can then be written as

131. $$H_{\underset{\sim}{k}} = \sum_{\underset{\sim}{k}} \varepsilon_{\underset{\sim}{k}} C^{+}_{\underset{\sim}{k}} C_{\underset{\sim}{k}} - V C^{+}_{\underset{\sim}{k}'} C^{+}_{-\underset{\sim}{k}'} C_{-\underset{\sim}{k}} C_{\underset{\sim}{k}}$$

and the approximate ground state wave function, Ψ_0, is

132. $$\Psi_0 = \prod_k \left(\mu_{\underset{\sim}{k}} + v_{\underset{\sim}{k}} C^{+}_{\underset{\sim}{k}} C^{+}_{-\underset{\sim}{k}} \right) \Psi_{vac}$$

where Ψ_{vac} is the Fermi sea vacuum state and where $\mu_{\underset{\sim}{k}}, v_{\underset{\sim}{k}}$ occupation numbers are constant. See equation (130). The product is taken over k states. In the ground state Ψ_0 all electrons are paired and $v_{\underset{\sim}{k}} = -v_{-\underset{\sim}{k}}$ because C^+, C anticommute or $[C^+, C] = -[C, C^+]$. The subspace in both states $\underset{\sim}{k}, -\underset{\sim}{k}$ of a Cooper pair are either both occupied or both empty. Recall that at the Fermi surface $\varepsilon_{\underset{\sim}{k}} = 0$ and the only field acting on a spin $\left|\underset{\sim}{k}\right| = k_F$ and ρ_F is the density of states at the Fermi level. In the region ω_D to $-\omega_D$ where the constant, V, represents an attractive term in the Hamiltonian, then ω_D is of the order of the Debye energy and the solution to the BCS energy-gap parameter Δ becomes

133. $$\Delta = \frac{\omega_D}{\sinh\left(1/\rho_F V\right)} \cong 2\omega_D e^{-1/V\rho_D}$$

for $1/V\rho_D >> 1$.

For the first approximation to the excitation spectrum, the energy $E_{\underset{\sim}{k}}$ is given as

134. $$E_{\underset{\sim}{k}} = 2\left(\varepsilon^2_{\underset{\sim}{k}} + \Delta^2\right)^{1/2}$$

where we consider only the positive square root. The minimum excitation energy is 2Δ. The superconductor thus has an energy gap which is detected by heat capacity curves of tunneling electrons through a barrier (Josephson junction) and transmission through thin films in far infrared radiation. As long as $V > 0$, the coherent state is lower in energy than the normal Fermi state, hence the criterion for superconductivity is that $V > 0$ and for nonsuperconductivity is $V \leq 0$. The critical fields at $T_c = 0^o K$ is $E_g = H_c^2 / 8\pi$ for a system of unit volume.

We can extend this formalism to the finite transition temperature in the theory of ferromagnetism. Thus if we relax our criterion that terms $a^+ a = 0$ and $aa^+ = 1$ for $T_c << \omega_D$, then down to $\varepsilon \approx T_c$ we can calculate approximately $T_c = 1.14\omega_D e^{-V\rho_F}$ and thus $2\Delta = 3.5T_c$ using our equation (133) for 2Δ. For example, $2\Delta / T_c = 3.5$ for S_n, 3.4 for Al, 4.1 for Pb, and 3.3 for Cd, and hence we observe the $T_c m^{1/2}$ isotope effect which is key to the observation of the existence of coherent states. This law may break down for higher temperatures, for $\mathrm{T} \gtrsim 70^0 K$. Neither V nor ρ_F depend on m, but ω_D is directly related to the frequency of the lattice vibrations, or phonon states in the vacuum or media. The frequency of an oscillator of a given force constant is proportional to $m^{-1/2}$ and thus $T_c m^{1/2}$ is a constant for an isotope variation of a given substance. (Well known exceptions are Ru and Os, both transition elements, where perhaps other particle coupling mechanisms occur.)

Another issue is the electrodynamics of superconductors. For gauge invariance, for the Coulomb gauge div $\underset{\sim}{A} = 0$ which is the usual form of Maxwell's equations, i.e., no divergent $\underline{B}$ fields or magnetic monopoles, then the normal state of paramagnetic current approximately cancels the diamagnetic current. The energy gap is zero in the normal nonsuperconducting state. For a normal insulator the vertical excited state is reached by a one-electron transition but for a superconductor only two electron transitions occur [23]. T.R. Schrieffer has demonstrated the gauge invariance of the BCS theory, and in ref. [24] he also examines plasmons in solid state plasmas. A gauge transformation from the Coulomb gauge implies that one adds to the vector potential, $\underset{\sim}{A}$, a longitudinal part $i\underset{\sim}{q}\phi\left(\underset{\sim}{q}\right)$ (where the expectation value of the diamagnetic current operator is taken for $q \to 0$). Such a term in the potential is coupled strongly to plasmons excitations and in fact shifts the phonon coordinates. This shift does not change the plasmon frequency and hence does not change the physical properties of the system. The gauge transformation is equivalent to the transformation $\Psi \to \psi e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}$ where $\underset{\sim}{K} = \underset{\sim}{k}_1 + \underset{\sim}{k}_2$. Note the assumption for such a gauge transformation is for Hertzian waves only. If we relax the gauge condition then the plasma coordinate shift may indeed occupy a physical role and can accommodate non-Hertzian phenomena, which occur in plasmas. In fact, coherent effects are a major feature of the experimental results of superconductors, superfluids, and plasma collective states, sometimes termed "instabilities." For superconductivity, there is remarkable and cogent evidence for the validity of the BCS formalism. Our two examples of dealing with the coherent phenomena of plasmon states and superconductivity seem well suited to the soliton model. It is through long-range collective states that these modes appear to act independently like a BEC condensate having a "mind of their own" [72].

Electron-plasmon interactions have been demonstrated to have soliton properties in dense media as we have formulated here. Applications of the soliton solutions apply to a number of hydrodynamic type media. We will consider a dynamic picture of unbound "free" electron coupling to phonon or plasmon acoustic modes. As we have seen in earlier work (see references 5 and 27), we can formulate the phonon modes in terms of collective vertical modes of vacuum state electron-hole excitations. This formalism describes modes of excitation of the polarizable and active vacuum [5,40-42]. We consider a number of systems which have long-range coherent phenomena such as a solid-state room-temperature system, comprised of a regular array of positive ions and band layer electrons. Let us assume that we can describe an effective array of ions by a spacing parameter ξ_n, where ξ_n is the n^{th} ion in the array [75].

We formulate the soliton model for a BCS type media. Let us consider a grand state ϕ_0 and a wave function $\psi_n(-1)^n \xi_n$ for the one dimension in space x. Then we can write an effective Hamiltonian as

135. $$H_{eff} = \frac{m}{2}\sum_n \dot{\xi}_n$$

where m is the effective soliton mass. The term $\dot{\xi}_n$ is the *pseudo-velocity*. Note that the use of a spacing parameter can be applied in a Monte Carlo random phase approximation to the approximately fixed ions of a plasma. Regular ionic spatial arrays can act as "wave guides" for standing longitudinal acoustic waves or as soliton transmission "conductors," whether coherent states in plasma, ordered liquid, or solid state systems.

Returning to our formalism of references [3,44,45] in which we express the coupling between degenerate states in terms of torque and Coriolis terms in the Hamiltonian, we introduce the Coriolis effect in the kinetic energy terms and the torque and spin angular momentum in the potential energy terms. The coupling term $g^2\left(\overline{\psi}\psi\right)$ of the Schrödinger equation, expressed with these considerations is given in terms of $\partial/\partial t$ where χ and τ represent modified metrical spacetime. The equation for one spatial dimension is given in terms of an interaction potential

136. $$V = g^2\left(\overline{\psi}\psi\right) + V(X,t,\tau).$$

For one spatial dimension

137. $$\frac{\hbar}{2m}\frac{\partial^2\psi}{2x^2} + g^2\left(\overline{\psi}\psi\right) + V(X,t,\tau) = \mathrm{i}\frac{\partial\psi}{\partial t}$$

for a potential free interaction and for effective-mass in the space with torque forces, m, and $\overline{\psi}$ is the complex conjugate of the wave function, ψ, giving the probability of $\left|\overline{\psi}\psi\right| = \left|\psi^2\right|$.

The diagonal element of the time evaluation operator in x,t space corresponding to $\eta_{\mu\nu}$ is given as

138. $$\Im = \langle x|e^{-t_{op}H}e^{-\tau_{op}H}|x\rangle = \sum_n |\phi_n(x)|^2 e^{-tH}e^{-\tau H}$$

where t_{op} and τ_{op} represent operators for t and τ and H represents the energy states of the system.

We expand the domain over which our operators can operate. This new space corresponds to our generalized metric

139. $$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_{\mu\nu} \, ,$$

and our new metrical space (X, χ, t, τ) accommodates the torque and Coriolis forces. The nonlinear term in the potential $g^2(\overline{\psi}\psi)$ incorporates the torque term and the term $V(X,t,\tau)$ incorporates the Coriolis effect. The solutions to our new equation are given in terms of a new symmetric harmonic oscillator. In fact, these solutions are soliton-like. We detail this model in the next section. The additional terms in the Hamiltonian operator are nonlinear and act to overcome dissipative losses in the system through energy dissipation. Hence, torquing stabilizes the system whether in a motor or in galactic structures. This continuous action of the expanded spacetime gives rise to the dynamical but stable structures we have observed in nature and can be constructed in laboratory [77-79]. Coherent states give rise to soliton solutions which do not destructively interfere and hence represent a manner in which coherent states generated under torque forces produce a balance dynamic in black hole physics. In such a system, the surrounding media and the central black hole form a constructive balance that allows stability to occur.

The lowest state ϕ_0 exists for the largest t where first order "classical" terms dominate. We have

140. $$\Im \lim_{t\to\infty} \to |\phi_0(x)|^2 e^{-tE_0} e^{-\tau H_0}$$

for $t_{op} = \mathrm{T}$ and $\tau_{op} = \Im$, where the subscript *op* stands for operator. We can now write an action variable associated with the usual space (x,t) and expanded space (χ,τ) time evolution operators, so that we have

141. $$\langle x | e^{-TH} e^{-\Im H} | x \rangle = \sum_{\lambda\delta} e^{-s\lambda} e^{-s\delta} \, .$$

The expanded x,t,τ "space" allows for collective coherent states operating over long distances and thus a mechanism for soliton creation and conduction in the media defining lattice-like vacuum structures. The terms $e^{-s\lambda}$ and $e^{-s\delta}$ are Boltzmann factors.

We can write the associated action variables in the linear approximation in which the coupling constant g^2 is small; that is the term $g^2(\overline{\psi}\psi)$ is small compared to the spatial or temporal terms dependency of the potential term $V(x,t,\tau)$. This is the case where the effect of torque is not so great. Then

142. $$S_{\lambda\delta} = \int_0^T \int_0^\Im \frac{m_\lambda}{2}\left(\frac{d\lambda}{dt}\right)^2 + \frac{m_\delta}{2}\left(\frac{d\delta}{d\tau}\right)^2 + \mathrm{v}(\lambda(t),\delta(\tau)) dt d\tau$$

where the potential $\mathrm{v}(\lambda(t),\delta(\tau))$ is associated with $V(x,t,\tau)$. The masses m_λ and m_δ are the associated effective-masses for the space (x,t) and space (χ,τ) action terms, respectively. Let us consider a simple example of a harmonic oscillator potential in one spatial dimension as a first order approximation, $V(x) = \frac{1}{2}kx^2$, for the spatial dependence of the potential. Consider also that the time evolution operator, τ, is proportional to finding a periodic path in the x direction in a equilibrium ensemble of all periodic paths in which each coordinate ξ_n relates to a static configuration of ions with two quantum degenerate configurations. The action terms s_λ and s_δ represent the energy of states λ and δ, respectively, as they appear in the Bolzmann factors.

Using the harmonic oscillator potential form $V(x) = -\frac{1}{2}kx^2$, and a dependent Gaussian distribution for $\phi_0(x)$, we can determine an expectation value for the x dependence of the particle coordinates α_n. This value is approximately the fluctuation range of the solitary wave. Slow variations in the ordered displacement within a crystal-like lattice correspond to long wavelength acoustic phonons. If we choose the phonon velocity to be of the order of the velocity of sound in the media, we find a size-scale expectation value of the solution to our nonlinear equation of $\langle \Psi \rangle \cong 0.04\,\mathrm{\mathring{A}}$ for a sixteen-site polyacetylene ring as an example. We determine that soliton or coupled phonon phenomena occur in the range of $0.3 \pm .01\overset{o}{\mathrm{A}}$. This size scale varies somewhat with a cut-off parameter which defines the spatial domain of the phonon. We can define the mean square size of the fluctuations such that they are less than $\left\langle (\delta\Psi)^2 \right\rangle$. This technique avoids logarithmic divergences in a structured array. This formalism leads to the concept that the vacuum itself acts as an ordered array of lattice-like structures.

The periodicity of the system, its degeneracy, as well as the large degrees of freedom afforded by highly resonant structures, may depend ultimately on a structured vacuum model. Frequency, geometric configuration, and size scale are key to the development of coherent states of excitation of the system. Let us consider some examples of exciton states and superconductivity and superfluidity.

In order to understand the structure of quantum processes in highly curved spacetime, we can take some clues from condensed matter physics. In particular, some of the coherent states that arise in ordered lattices, such as phonons, may well describe the manner in which the vacuum acts as a medium. In reference [50], one of us (Rauscher) has demonstrated that although Einstein, in his relativity theory, believed there was no preferred frame of reference, the structure of the theory of relativity is consistent with a preferred frame of reference such as indicated by Mach's principle. This picture of the quantized nature of spacetime acting as a "molecularized" granular fluid is consistent with the Heisenberg Uncertainty Principle and vertical pair production and vacuum state polarization from a non-empty vacuum. In the same body of work [50] an Ether concept is reintroduced as a neo-Ether having properties that support a quantum gravity model in a multidimensional geometry. Properties of this neo-Ether are identified with the properties of dynamic polarizable vacuums that support collective, coherent states of matter. In references [25,44,49,50] spacetime appears as an approximately continuous geometry, but at the quantum scale spacetime becomes "grainy" or structured, that is, quantized [25,44]. This picture yields an analogy between a vacuum "neo- Ether" which supports collective state phonon-like modes in a discrete fluid-like medium.

The other of us (Haramein) has introduced the concept of a structured vacuum arising from a spacetime torque and Coriolis effect driving specific coherent states of matter. Essentially, in analogy to a molecular fluid made up of discrete atomic or molecular "grains," the hydrodynamic and charged structure of a spacetime manifold incorporating torque and Coriolis dynamics results in fluid structures with quantized discreteness [3]. This latter model acts as a "condensed matter state" arising from the lattice of a polarized structured vacuum, [47] where the "Ether-like" functions of spacetime are in fact a direct consequence of a driving torque producing solitons/phonon eddy-like structures defining granularity, such as the Planck quantities.

The startling and profound results of the recent Brookhaven Relativistic Heavy Ion Collider (RHIC) accelerator yields a surprising picture of a superfluid state of matter arising from the vacuum state energy structure, which is consistent with early Universe conditions and black hole physics [35,36] — giving us experimental verification of our theoretical model [65]. The phonon state arises when the internal agitation of molecules in the media are slowed, such as for low states near absolute zero, as in interstellar space. Phonons arise in crystal lattice structures and in low temperature fluids as experimentally observed in the laboratory. In a fluid which is moving in a non-uniform manner, the phonon velocity of propagation varies with the wavelength as $\mathrm{v}_s \approx \lambda \dfrac{\omega}{2\pi}$. A parallel can be made between photons in a curved spacetime and phonons in an inhomogeneous medium.

An analogy is made by one of us (Haramein) describing sound waves in a stream of water entering the constriction of a drain. The acoustic phonons become distorted and follow the bending path, like light photons around a massive body such as a star. The fluid flow can act on sound in the same manner as a black hole does on light. In this analogy, it is clear that rotation occurs as the fluid enters the drain and so does light crossing the event horizon. Hence spacetime is not only quantized and granular, but also is torqued or under the influence of spin. In fact, it is the vacuum acting on the black hole from the torquing of spacetime-matter/energy that rotates or spins the black hole and expands the event horizon into an ergosphere. Further, the dynamic exchange between water and air (air is ejected to allow water to go down) in our "drain" analogy is the basis for our balance equation and is an analogue to the Coriolis energetics at the surface of the horizon. The recent data on fireball collisions at BNL's RHIC and Hawking's model support the fluid dynamic picture of the vacuum media as do the collective states developed in the plasma medium. In fact, all these coherent states of excitation in lattice structures, cool fluids, and plasma gases arise from the vacuum structure itself. At the interface of the horizon of black holes, the deepest and clearest manifestation of the polarized vacuum structure occurs.

One of us (Rauscher) had proposed in the early 1980s that there is a fundamental connection between the collective states that occur in plasma electron state dynamics and the collective electron states in superconductivity [72]. Although laboratory applications of this theoretical construct have not been made, it is only recently that the

CERN-RHIC and BNL-RHIC work has yielded an extremely fascinating application of the earlier theoretical work, which we have greatly expanded upon in this paper [65,66,68]. Collective state flows in heavy ion high energy $400 GeV$ collisions have produced fluid dynamic-like quark-gluon plasma states [72,80-82]. Superdense hydrodynamic high viscosity states are formed which also exhibit Lattice QCD phase-like transitions [19,47]. These superdense states, of course, occur under strong gravitational conditions in the vicinity of a black hole. At the quantum extreme, the fluidity becomes granular at the Planck scale where the full quantum treatment is required.

Photons produced near a black hole have very short wavelengths resulting in experiencing the granularity of the fluid properties of the event horizon. Four major recent analogues of the properties of black holes have been made: first, the event horizon of the black hole carries phonon-like excitations; second, the properties of the event horizon act like superfluid helium with near zero internal resistance; third, electromagnetic phonon excitations occur at or near the event horizon; and fourth, Hawking-like radiation [35,37]. We can treat the black hole as a black body radiator where light quanta are emitted by harmonic oscillators as photons. The standard solutions to the Schrödinger equation are harmonic oscillator solutions. In the Hawking radiation model, photon excitation of pair creation and absorption is produced as the lowest state oscillation of the vacuum. The fact that photon emission occurs in Hawking radiation and from a black body radiator may allow us to better comprehend the physics and quantum effects at or just externally to the event horizon.

The soliton model may be a viable approach to explain the coherent states that allow the torquing mechanisms to maintain coherence in galactic structures. However, there is no reasonable manner to account for the existence of collective coherent phenomena, such as plasmons, phonons, acoustons, and excitons in apparent "free space." In order to account for such phenomena, one needs to consider the origin of phonons and other coherent states in conventional physics. Such states normally arise in lattice structures which are set into vibratory modes, for example, sound phonon vibrations in structured or crystalline matter. Thereafter, by pursuing this manner of reasoning we can only conclude that, since these collective coherent states arise in the vicinity of a black hole, that the space in this region acts as a lattice structure. The Lindquist-Wheeler [19] approach appears most relevant in our consideration of the structure of black holes and their surrounding media.

Our approach to formulating the nature of black holes in astrophysical and cosmological space and their surrounding media dramatically demonstrates the possibility of deducing the detailed properties and structure of the vacuum. The collective, coherent modes of excitation and oscillation of the media surrounding black holes can only occur if they are structured by the excitation of the vacuum. These states and others are manifestations of the photon equivalent of phonon vibrations in a crystal lattice. We will now deduce the nature of that structure which seems to relate the double torus $U_2 \times U_2$ to a cuboctahedron and S_4.

X. DEDUCING PROPERTIES OF THE STRUCTURED VACUUM

From the results of our previous calculations, we are driven to the inevitable conclusion that the collective, coherent modes of plasma oscillations can only be supported by a structured vacuum. In this section, we discuss the evidence for a structured vacuum and the manner in which both the laboratory and astrophysical observations imply a particular structure of the vacuum as the most viable model.

Stable packets of charged particles moving collectively through spacetime is fundamental to plasma structures. These states occur through localized electromagnetic wave phenomena and can persist like solitary wave phenomena for a fairly long time. We have treated the particle coherent states or particle packet as warm or hot electron plasmas, analogous to a fluid state discussed in the previous sections. Plasma and fluid dynamics appear to be close allies in laboratory and astrophysical systems and applications. Each specific collective mode that we have formulated and discussed corresponds to specific velocities of propagation in the ionized media around the black hole event horizon. These modes act like indicators of the structural forms of the vacuum. Each one of the collective, coherent modes, such as plasmon, acouston, phonon, exciton, and Bose–Einstein condensate, have a specific mode of propagation. These relative propagation velocities are clues to the fact that the vacuum contains properties of a lattice structure. Each propagation velocity indicates a sub-structure of these lattice forms. These calculations indicate a set of lattice structures and sub-structures. We can deduce the form and nature of these lattice structures from the velocities of propagation of the collective, coherent states they sustain. Also, these lattice structures can be identified in the group theoretical framework.

Some of the respective velocities of propagation of the collective states which are sustained by the medium surrounding black holes are the plasmon at about $10^5 cm/\sec$, the acouston at $10^4 cm/\sec$, the phonon at about $10^2 cm/\sec$, and the BECs which only propagate locally, condenses and exchanges energy with the media, and then recondenses. These velocities are somewhat temperature-dependent. The temperature in the media outside the ergosphere is from $10^{4\,o}K$ to $10^{5\,o}K$. Closer to the ergosphere the particle excitation energy increases and hence the temperature becomes higher. These temperatures can rise to $10^{10\,o}K$ near the horizon. Acoustons can exist in temperatures up to $10^{5\,o}K$.

In current cosmology, it is believed that at least 25% of all energy comes from black holes. The mistaken view is that black holes only consume energy and matter, but, in fact, they act like generators which excite the surrounding media, generating energy as heat. Giant black holes such as those near galactic centers of the order of $10^6 M_\odot$ or greater can also lead to stellar production. It is clear that black holes drive vast amounts of energy through them and contribute vastly to the dynamics of astrophysical systems. In fact, black holes may be the manifestation of the driving forces of spacetime torque and Coriolis torsional forces in the vacuum, expressing itself as galactic, stellar or even atomic structures. Thus, black holes may contribute to a large portion of the "missing mass" when accurate accounting has been made for the drive mechanisms necessary to produce the angular momentum/spin observable at all scales.

Dressler analyzed the evidence for massive black holes at galactic centers. The acceleration of gravity is, of course, greatly increased in the vicinity of the massive and unique objects in our universe [83,84]. Careful observation by Dressler found that stellar objects near galactic centers moved around the center at $150{,}000 - 300{,}000 km/\sec$ with closer ones moving more rapidly in their circular orbit. Careful analysis demonstrated the best interpretations of the results strongly indicate the presence of galactic center black holes including an approximately $3\times10^6 M_\odot$ massive black hole at the center of our Milky Way. Short γ -ray bursts are observed daily and are thought to occur from supernovae explosions with jet propulsion on top and bottom perpendicular to their plane. Pulsars are thought to be the end product of supernovae production. Supernovae also produce visible, UV , x -ray and γ -ray, emissions. We believe that supernovae structures are a major clue as to the dynamics of cosmological phenomena. In fact, these structures display the geometric form of matter fundamentally affected by the vacuum. Galactic formation and many other astrophysical structures are manifestations of the underlying vacuum structure and energy-driven processes.

In terms of coherent states which allow long range effects to occur that allow for the transmission of information over long distances, we have considered soliton states in plasmas, and we can consider the BEC formation in more detail. The BEC states also exhibit polyhedral configurations, thus information can be synchronized between the black hole external plasma field where $r > r_H$ and the internal black hole states for $r \leq r_H$ where r_H is the radius of the event horizon. Bose–Einstein condensates (BEC) have been found to act like a soliton and move relatively long distances without spreading out. Strecker, et al. [85] and Khaykovich, et al. [86] have conducted experiments with lithium, Li^7 , atoms, adjusting the inner atomic distances in tunable magnetic fields from repulsive forces to form a stable BEC, having weakly attractive forces. Wave packet dispersion is overcome by the self-focusing nonlinearity to form solitons.

In this section, we discuss the structure of the vacuum and the manner in which standing waves are set up and sustained in the media. It is clear that the coherent states that arise in the plasma media fundamentally depend on an underlying and periodic structure. This structure we identify as inherent in terms of the vacuum. If we define the Dirac vacuum as $|0\rangle$ the Fermi–Dirac model is relevant to the interpretation of plasma phenomena. The properties of the plasma depend on a structured polarizable vacuum which implies a spacetime metric that has an inherent local asymmetry and has global symmetry. The asymmetry is expressed not only by the structure of the vacuum but by the force torquing of the metric of the modified Einstein field equation with torque and Coriolis forces [3,44].

As the torquing properties of the metric are considered, in this case the Haramein-Rauscher metric, the vacuum becomes polarized and carries a fundamental spin-rotation. It is clear from the sustained collective plasma states that the biased vacuum is not only structured, but that it must also necessarily be dynamic. Coherent plasma states could not exist as localized waves due to nonlinear effects unless these nonlinearities and polarization properties existed within the vacuum structure itself. The so-termed fourth state of matter or plasma is the most plentiful state of matter in the Universe, occurring in cosmological interstellar media, and in stellar, supernovae, and black hole structures and other astrophysical features. The properties of plasma media most directly display the structure of a vacuum.

In reference [3], we have demonstrated that the double torus $U_2 \times U_2$, resulting from an addition of spacetime torque and Coriolis effects in an Einsteinian metrical space, is fundamentally related, through the 24-element octahedral groups, $C[\overline{O}]$ and $C[O]$, to the dual symmetry operations of the S^4 groups and the tetrahedron under the A_4 group. The dual torus dynamics are prominent across scales from quasars with their jets, galactic halos, black holes ergospheres, supernovae, to solar and planetary plasma phenomena. Furthermore, there seems to be evidence of tetrahedron/octahedron vacuum lattice structures defining large scale supercluster arrangements [87,88], supernovae internal plasma behavior [44], and planetary bands and vorticular energy events in gaseous planets which correlate well to these specific angular relationships, such as the bands on buth Jupiter and Saturn [89]. Recently, data returned from the Cassini probe confirmed the 26 year old earlier imaging by Voyager 1 and 2 of the presence of a persistent hexagonal feature on the north pole of Saturn [90]. The enormous vortex, approximately 15,000 miles across, exhibits highly geometric boundary conditions where winds traveling at nearly 350 mph are turning the corners of a very well defined hexagon [90]. This feature and the recent discovery of a southern pole vortex extending deep into the gaseous interior of the planet, combined with clearly delimited banding divisions at latitudes that correlate well with an hexagonal $8 \times 8 = 64$ cuboctahedral matrix, may be further evidence of the tetrahedral/octahedral polarized vacuum structure producing coherent collective behavior in the plasma of a torquing toroidal metric [3].

The cube and octahedron are dual to each other under the operations of the S^4 group and the tetrahedron is dual to the cube under the A_4 group. Note also that the icosahedron and dodecahedron are dual under the A_5 group. The simplest and lower dimensional group is the 24 elements of the octahedral group, O, rather than the 60-element icosahedral group, I. Hence, the most basic vacuum structure which can generate harmonic oscillator solutions is the cuboctahedron, which is the only geometry directly mappable to the double torus, $U_2 \times U_2$, having four copies of U_1. The movement from cuboctahedron to a pair of interpenetrating polarized tetrahedra (stella octangula) acts like a pumping action moving through rotation, from one state to another. This structure can exist in two extreme conditions: the cuboctahedron, which can collapse in orthorotation passing through the icosahedron and the octahedron to eventually reach the stella octangula [91] or dual tetrahedron, and back in harmonic oscillations.

In terms of harmonic oscillation solutions, we require a potential, V, and which can be expressed as a gradient so that $\nabla^2 V = 4\pi\rho_0$. In this case the density is associated with $|0\rangle$ as a vacuum density expressed as a gradient of a potential. The potential, V, is the potential in the Hamiltonian, $H = T + V$, for kinetic energy, T. As the cuboctahedron maps into the tetrahedron, the S^4 group is mappable to the A_4 group and oscillates in the two extreme modes in a harmonic oscillator motion. This motion works as a torquing pumping action in which the vertices of the cuboctahedron to the stella octangula and back form a phi spiral motion (resulting from the icosahedron relationship) reaching the end points of S^4 and A_4. These end points are analogous to the end points of a pendulum which undergoes simple harmonic motion under gravitational potential. At the end points of motion, the system comes to rest at one vector equilibrium state and then proceeds to the other vector equilibrium state associated with the point of the pendulum motion where $H = V$. At the midpoint of the cycle, when the pendulum is vertical to the gravitational field, maximum kinetic energy states exist for $H = T$. This repeated cycle of oscillation generates harmonic oscillator state solutions.

The classic paper on the lattice cell universe by Lindquist and Wheeler [19], suggests that the homogenous isotropic closed Universe model be replaced with a Schwarzschild lattice closed Universe model. In the former case, the mass of the Universe is distributed uniformly and in the latter case, the mass is concentrated into 120 identical Schwarzschild black holes, each located at the center of its own cell. Each cell is a dodecahedron bounded by 12 faces, each approximately a pentagon. Note that the dodecahedron is dual to the icosahedron under the A_5 group and that in our case the icosahedron is generated during one of the intermediary states of the cuboctahedron cycle of oscillation. In the model of Lindquist and Wheeler, many Schwarzschild zones are fitted together to comprise a closed Universe which is dynamic in that a test particle at the interface between two zones rises up against the gravitational attraction of each zone and falls back under the gravitational attraction of each zone. Therefore, the two centers themselves must move apart and back together again in sort of a breathing motion. This occurs for all pairs of centers, thus the lattice Universe itself expands and contracts, although the Schwarzschild geometry is viewed as static. Lindquist and Wheeler, approximate each lattice cell as an idealized sphere to simplify their analyses, in the same manner utilized in solid-state physics. In this approximation, the geometry inside each lattice is dealt with as a Schwarzschild sphere. This system is treated as expansion and re-contraction as independent Schwarzschild cells unless they come too close together to coalesce. We have also considered a similar but unique crystal lattice model to explain the support of the plasma coherent states [20,47]. The maximum radius is $a_0 = \frac{4GM}{3\pi c^3}$ and the relation between radius, a , and co-time, T , is $a = cT$ and is cycloid of a given parameter

143. $$a = \left(\frac{a_0}{2}\right)(1+\cos\eta) \text{ and } T = \left(\frac{a_0}{2}\right)(\eta + \sin\eta)$$

where the authors let $r \equiv a\sin\chi$ define χ and $\eta \equiv \left(\frac{2T}{a^*}\right)^{1/2}$. The (χ,η) are their "coordinates" for a phonon's path, which is a geodesic in the Lindquist–Wheeler cell. The number of zones, N , is given in terms of the hypersphere solid angle of the entire 3 sphere equals $2\pi^2$ and the hypersphere solid angle of one zone, $\frac{4\pi}{3}\chi^3$ so that for a total solid angle of $8/_3\,\pi^3\chi^3 = \frac{8}{3}(\pi\chi)^3$.

In addition to the structures of individual systems, there have been very interesting surveys conducted on the distribution of superclusters which displays a remarkable periodicity. Battaner [87] and Battaner and Florido [88] have considered the large scale structures of the order of $100Mpc$ which is the deepest survey with a resolution to $10Mpc$. They observe a network array of galactic clusters. Systematic statistical analysis indicates a strong probability that these clusters form an octahedral array in which the octahedrons are in contact at the vertexes and thus creating cuboctahedrons. It is believed that the magnetic fields of the radiation dominated Universe comprises a network of filaments produced by early large scale magnetic fields and may have produced the octahedral array. The observation of these arrays appears to be more fundamental than pattern recognition, as they are such dominant features observed in the deep survey [92]. Then we have,

144. $$N = \frac{2\pi^2}{\frac{4\pi}{3}\chi^3} = \frac{3\pi}{(2)^{5/2}}\left(\frac{a^*}{T}\right)^{3/2}.$$

The Schwarzschild cell method predicts a cycloidal relationship between radius of the Universe and proper co-time, T , which was formulated by Friedmann [93]. Then $r = a\sin\chi$, see equation (143). A path on a 3-sphere is given as $\eta = \left(\frac{2T}{a^*}\right)^{1/2}$. The relationship between the Lindquist and Wheeler Schwarzschild sphere and the vertices of the Battaner and Florido regular geometric structure of superclusters can be compared. For N vertices, each vertex can be equidistant from its nearest neighbor only when N =5, 8, 16, 24, 120, or 600 [94]. The case where $N = 8$ yields the simplest arrangement. In this lattice, N =5, 16, and 600 correspond to a tetrahedron, N =8 to a cube, N =24 to an octahedron, and N =120 to a dodecahedron. Correspondence is made in terms of the ratio of the distance from a face to a corner of a cell of some volume of a regular polyhedron to a sphere.

One of us (Rauscher) [25] treated the whole Universe as expanding under a Schwarzschild condition. We found that consistence between Einstein's field equations with big bang cosmologies can be obtained but requires the introduction of an additional term in the stress-energy tensor. We can associate this term with the torque term in Einstein's field equations in the Haramein–Rauscher model [3]. One of us (Haramein), has put forward the need to include spin and torque to modify the simplistic Schwarzschild metrical zones of Lindquist and Wheeler although their model is very useful in our considerations even if it is clearly a limited case.

The motivation of the Lindquist and Wheeler model is that the cell method in gravitational theory contains a new dynamic feature which expresses the equation of motion of a mass at the center of a cell as a dynamic condition on the boundary of the cell. The boundary condition defines a constraint on the space which comprises simple geometric forms. The whole of the dynamics of this model are expressed in terms of the expansion and subsequent contraction of the Schwarzschild solution to Einstein's field equation. Their analogy is to that of a crystal lattice and by defining cells in terms of a Schwarzschild solutions in a curved space, in a simple Friedman metric of uniform curvature which corresponds to a polyhedron in Euclidian space. They derive a boundary condition on the Schwarzschild potentials which do not go to zero at a finite radius and hence avoids the discontinuity of matching the normal derivative of the gravitational potentials which would occur in the Schwarzschild solution alone. In the lattice Universe, mass is concentrated into N centers (or vertices) which could correspond to the galactic cluster centers in the Battener and Florido analysis [87,88]. In each cell, a Schwarzschild black hole is located at the center of its own cell. In their figure 3, six cone shapes define their boundary conditions in a lattice Universe and correspond to the vertices of an octahedron. Therefore, a parallel can be made between the work of Lindquist and Wheeler, Battener and Florido and our model which predicts a polarized structured vacuum. Hence, Lindquist and Wheeler's approach using the Schwarzschild cell solution without spin or charge gives a good first-order approximation. We use the Kerr-Newman with spin and charge and incorporate the torque and Coriolis forces in the Haramein-Rauscher solution to quantize the vacuum into cells.

We consider the topological structures of the current string theory and our approach to the unified theory of the four forces and structured vacuum [3]. Although superstring theories have their critics, due to the fact that those theories contain a number of "free" parameters, there has been great interest in these theories by the physics community. Superstring theory has been related to the standard model. Some string theories contain gravity and others do not. One of the major features of superstring theory is to treat particles as tiny loops rather than as point particles so as to avoid the problem of singularities. The string theory approach has some topological similarities to that of Lindquist and Wheeler's work, which is an effort to avoid singularities. In the string theory, particles are treated as vibrations of a membrane (Brane $M-$surface), which is swept out by the vibrating string occurring in eight dimensional space. These eight dimensions comprise eight of the ten dimensional standard model in which two of the dimensions are the string surface itself. This vibrational space carries the symmetry of the Lie group E_8 [95]. Superstring theory represents both bosonic and fermionic particle states. The usual string theories occupy a 26-dimensional spacetime, representing bosonic particle states. A quantum state of identical bosonic particles is symmetric under the exchange of any two particles. A quantum state of identical fermionic particles is antisymmetric under the exchange of any two particles to include the photon and gravitation. Then we have $8\times 8 = 64$ dimensional states in some superstring theories. The closed string theory is called a type II string theory, which has the doubly fermionic states included, for a total of $2\times 8\times 8 = 128$ fermionic states [96].

In addition to the type II, there are two heterotic superstring theories which involve closed strings. Out of the 26-L bosonic coordinates of the bosonic factor, only ten are matched to R-bosonic coordinates of the superstring factor, hence this theory effectively exists in ten-dimensional spacetime. Heterotic strings comes in two versions, that is $E_8\times E_8$ and the $SO(32)$ type. The Ramond vacuum is included and E_8 is the highest dimensional exceptional group. The $E_8\times E_8$ superstring theory is derived from the compilation of $M-$theory. One of the most promising superstring theories that unifies the four forces is the $E_8\times E_8$ reflection space. This is possible only because reflection embedding provides for an embedding of A_4 in E_8 [97]. In our paper reference [3] we present the symmetry group relationship between A_4 and the 24 element octahedral group. This procedure operates along the lines of the relationship between the $SO(32)$ heterotic string theory which also utilizes the $E_8\times E_8$ formalism. However, we believe our approach to gravitation and strong interactions, which considers the inclusion of torque and Coriolis effects will result in a simplification and a more fundamental formalism with fewer free parameters.

In general, the Lie algebra A_n associated with a reflection space C^n has a compact Lie group SU_{n+1}. S.P. Sirag attempts to develop a unified field theory in terms of $U_1\times SU_2\times SU_3\times SU_4$ where he identifies the SU_4 group with the tensor gravitational field [98]. Note that gravity is missing from the SU_5 theory. The $SO(32)$, or SO_{32}, is the group generated by 32-by-32 matrices that are orthogonal. For the strong force, gluons are described by a four dimensional SU_3 Yang–Mills theory. The full set of standard model gauge bosons is described by the Yang–Mills theory with the gauge group $SU_3\times SU_2\times U_1$. Alternatively, for the $U_5 = SU_2\times SU_3$ Yang–Mills theory, the gauge group that emerges as $U_3\times U_2 = SU_3\times SU_2\times U_1\times U_1$ where $U_1\times U_1$ is the topology of the torus. Note that the A_4 group of the tetrahedron is the label for a complex Lie algebra whose compact Lie group is

SU_5 which comprised the first unification, GUT theory. The standard force bosons are derived from the group $U_1 \times SU_2 \times SU_3$ in the group algebra.

In the heterotic $E_8 \times E_8$ superstring theory, six of the nine spatial dimensions are curled up into a small six-dimensional compact space, which is termed the Calabi–Yau space. All Calabi Yau spaces have both discrete and continuous parameters which determine the details of the four-dimensional theory that arises upon compactification. For all Calabi–Yau spaces, the minimal amount of supersymmetry survives the compactification and the resulting four-dimensional theory is supersymmetric. The compactification also allows one to break the original gauge symmetry $E_8 \times E_8$ down to $E_6 \times E_8$. The group E_6 contains $SU_3 \times SU_2 \times U_1$ as a subgroup to that standard model gauge group. An alternative to the 6-dimensional space compactification of the heterotic string is an alternative 6-dimensional space where one can simply use a six-torus T^6 group space. The T^6 space, however, has singularities that arise at the fixed points of certain identifications, but orbitals constructed from tori are much easier to analyze than the general Calabi–Yau spaces.

For the following Lie group $S = U_2 \times T^6$ where U_2 is a four dimensional spacetime called the conformally compactified Minkowski space and $T^6 = U_1 \times U_1 \times U_1 \times U_1 \times U_1 \times U_1$, or a 3-torus. We regard SU_2 as a spherical three space, S^3, as the usual space of cosmology. For a 7-torus T^7 which incorporates U_1 from the U_2 space also includes time. The T^7 tori space corresponds to the 7-reflection space E_7 because $T^7 = R^7/L$ where R^7 is the real part of the E_7 which also contains the complex reflection space C^7, and L is the root of E_7. This means that all parts of the lattice are identified as a single point: the identity element of T^7 and every other point of T^4 is a copy of L. The T^4 group can be identified with two double tori. We have identified the double torus structure as fundamental to a metric of spacetime which appropriately accounts for the source of spin/angular momentum. Many striking examples of this dynamic structure are observed at the cosmological scale such as galactic halos, black hole ergosphere and supernovae.

The S^4 group is associated with the 24 element octahedral group $C[\overline{O}]$ which can be written in terms of $C[\overline{O}] = U_2 \times \tilde{U}_2 \times U_4$ or T^8 group [3]. Both $C[O]$ and $C[\overline{O}]$ relate to the T^4 double torus group of four copies of U_1 where T^n is the direct product of n copies of U_1, which comprises the n torus, which is always an Abelian group. The T^n group refers to the structure of spacetime. We have related this spacetime structure to the torque term in Einstein's field equations [3]. Hence, the torus topology can be considered fundamental to the structure of spacetime and also the tenets in the superstring theory.

Hull utilized string theory in a "T-fold-background" with local $n-$torus fabrication and $T-$duality transitions functions in $O(n,n;Z)$ in an enlarged space with T^{2n} fabrication geometry [99]. For a geometric background, the local choice of T^n fit together to give a spacetime which is a T^n fiber bundle. Thus this string theory approach involves diffeomorphisms and gauge transformations as well as duality transformations. The $T-$duality is associated with mirror symmetry [100]. In some cases, the compactifications with duality are equivalent to asymmetric orbits. The full transition functions for the torus bundles, which are considered in Hull's approach, are in $GL(n,\mathbf{Z}) \times U_1^n$ where U_1 acts as a translation on a circle fiber. String theory compactification of dimensions on the T^n has $O(n,n;Z)$ symmetry. In the geometric $GL(n;\mathbf{Z})$ subgroup that acts through T^n diffeomorphisms, can be lifted to a higher dimensional theory which is compactified on a T^n fiber bundled over a circle. A $T-$duality on any circle gives a twisted reduction on a T^2 fiber bundled over a circle in $GL(2;\mathbf{Z})$ which is representative of a dual torus. These mirror, or duality symmetries are related to space with Calabi–Yau fibrations in space with torus fibrations [99]. The topology of $T-$folds, and their doubled formulations, is then seen as a geometric background in which there is a global polarization. The polarization can be characterized in terms of a product on the T^{2n} fibers. Local product structures satisfy integrability thus eliminating the problems of singularities. A product structure defines a splitting into eigenspaces of R with eigenvalues ± 1 and for a torus

T^{2n}. This extends to a splitting as the periodic torus coordinates into two T^n eigenspaces, if the product structure is integral, or $R \in GL(2n,\mathbf{Z})$, so that it acts on the coordinates while preserving the periodicities. A product structure and pseudo-Hermitian $O(n,n)$ invariant metric are together preserved by the subgroup $GL(n,\mathbf{R}) \subset O(n,n)$ and for the transformations acting on the torus and is preserved by $GL(2n,\mathbf{Z}) \subset O(n,n;Z)$ [3,5,20,47]. The fundamental structures activated in the vacuum by polarized coherent resonant states of matter also act as part of the process that creates these vacuum properties. To paraphrase John A. Wheeler, "Spacetime is not just a passive arena for doing physics, it is the physics" [2]. The torquing of spacetime is an active part of the structure of the stress-energy tensor and hence is a fundamental force coupling to produce the observable universe of matter and energy.

CONCLUDING REMARKS

We have a vast new set of tools to comprehend the processes of astrophysical and cosmological phenomena, atomic and collective matter states. For example some of the collective state phenomena we have considered are accelerator "fireballs," Bose–Einstein condensates, Fermi electron states, MHD and BCS descriptions, all of which obey soliton dynamic solutions. Theoretical and experimental findings and relativistic formulations, quantum theory, electromagnetic interactions can well be described in terms of topological structures and group theory. The fundamental base of our approach is to consider that the topological structure of a torquing spacetime, and its Coriolis gyroscopic dynamics, has critical aspects of unification theory.

We pursue this point further in references [39,101,102] when we consider atomic, nuclear, and quantum physics in a nonlinear space. When a torque and Coriolis term is considered for the formation of spin/angular momentum we find that the dual torus topology occupies a fundamental role in both astrophysics and quantum particle physics. The Haramein–Rauscher approach takes spin and rotation properties as fundamental to the structure of the spacetime manifold. We have identified the properties of the structure of the vacuum itself from fundamental coherent polarized states of matter in the facility of astrophysical black hole event horizons. That is to say, we have demonstrated that the properties of matter in superclusters, galaxies, supernovae and their vicinities, for example, could exist in resonant states, only if the vacuum is structured. These considerations may also be utilized to explain the effects that are currently attributed to dark matter and dark energy.

In the words of Nobel laureate C. N. Yang, of the Yang-Mills equation "*Einstein's general relativity theory, though profoundly beautiful, is likely to be amended... that the amendment may not disturb the usual test is easy to imagine, since the usual tests do not relate to spin... somehow* (the amendment) *entangles spin and rotation*" [103].

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